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A free boundary problem describing S–K–T competition ecological model with cross-diffusion

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ABSTRACT

In this paper we investigate a free boundary problem describing S–K–T competition ecological model with two competing species and with cross-diffusion and self-diffusion in one space dimension, where one species is made up of two groups separated by a free boundary, and the other has a single group. The system under consideration is strongly coupled and the coefficients of the equations are allowed to be discontinuous. We first show the global existence and uniqueness of the solutions for the corresponding diffraction problem by approximation method, Galerkin method and Schauder fixed point theorem, and then prove the local existence of the solutions for the free boundary problem by Schauder fixed point theorem.

1. Introduction

Consider a competition ecological model with two competing species and with cross-diffusion and self-diffusion. Assume that the first species is a kind of animal and the two species share the same habitat, the one-dimensional domain $(0, L)$, and further that the first species is made up of two groups, whereas the second species has a single group. Since the two groups of the first species have different biological habits and diffusion coefficients, then they do not live together and are separated by a free boundary $x = s(t)$ (see [1]). In this paper, we consider the situation where the cross-diffusion pressure for the second species is zero.

Let $u = u(x,t)$, $v = v(x,t)$ be the population densities of the two species. For $T > 0$ and for a given continuous curve $x = s(t)$, $0 < s(t) < L$, set (see Fig. 1)

$$I := (0, L), \quad Q_T := I \times (0, T], \quad \Gamma_T := \big(\{0, L\} \times [0, T]\big) \cup \big(I \times \{0\}\big),$$
$$Q_{1,T} := \{(x,t) : x \in (0, s(t)),\ t \in (0, T]\}, \quad Q_{2,T} := \{(x,t) : x \in (s(t), L),\ t \in (0, T]\}.$$

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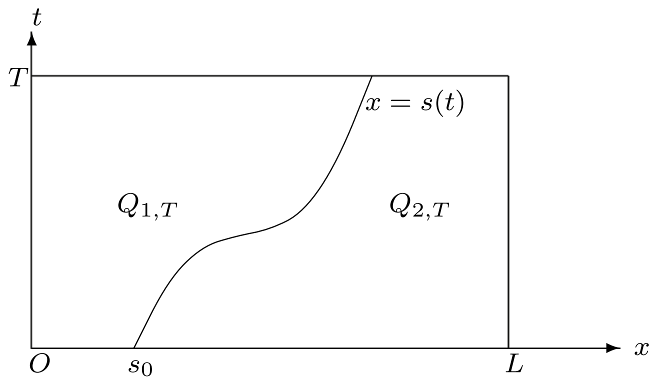

Fig. 1. The (x,t)-domain for the problem.

According to the result of Shigesada, Kawasaki and Teramoto [2], the flux of the population density can be represented as

$$J_1 = -\big((k_1(x,t)+\tau_1(x,t)u+\rho_1(x,t)v)u\big)_x \quad ((x,t)\in Q_{1,T}\cup Q_{2,T}),$$
$$J_2 = -\big((k_2+\tau_2 v)v\big)_x \quad ((x,t)\in Q_T),$$

where functions $k_1(x,t)$, $\tau_1(x,t)$ and $\rho_1(x,t)$ are defined by

$$k_1(x,t) := \begin{cases} k_{11} & (x\in[0,s(t)], t\in[0,T]) \\ k_{12} & (x\in(s(t),L], t\in[0,T]), \end{cases} \qquad \tau_1(x,t) := \begin{cases} \tau_{11} & (x\in[0,s(t)], t\in[0,T]) \\ \tau_{12} & (x\in(s(t),L], t\in[0,T]) \end{cases}$$

and

$$\rho_1(x,t) := \begin{cases} \rho_{11} & (x\in[0,s(t)], t\in[0,T]) \\ \rho_{12} & (x\in(s(t),L], t\in[0,T]), \end{cases}$$

and where constants k_{1i}, k_2 are all positive, and constants ρ_{1i}, τ_{1i} and τ_2 are all nonnegative. Then $k_1(x,t)$ and k_2 are the diffusion rates of the two species, $\tau_1(x,t)$ and τ_2 are their self-diffusion rates, and $\rho_1(x,t)$ is the cross-diffusion rate of the first species. In this paper, we further assume that there exist nonnegative constants a and b, such that

$$\tau_{1i} = ak_{1i}, \quad \rho_{1i} = bk_{1i}, \quad i = 1,2.$$

Besides, we denote τ_2/k_2 by e.

From the principle of conservation we see that the vector function $\mathbf{U} = (u,v)$ and the curve $x = s(t)$ are governed by strongly coupled system of equations in the form

$$u_t = \big[k_1(x,t)\big((1+au+bv)u\big)_x\big]_x + uf_1(x,t,\mathbf{U}) \quad ((x,t)\in Q_{1,T}\cup Q_{2,T}), \tag{1.1}$$
$$v_t = \big[k_2\big((1+ev)v\big)_x\big]_x + vf_2(x,t,\mathbf{U}) \quad ((x,t)\in Q_{1,T}\cup Q_{2,T}), \tag{1.2}$$

where $u_t := \partial u/\partial t$, $u_x := \partial u/\partial x$, and the functions $f_l(x,t,\mathbf{U})$ ($l = 1,2$) are defined by

$$f_l(x,t,\mathbf{U}) := \begin{cases} f_{l1}(\mathbf{U}) = r_{l1} - \beta_{l1}u - \gamma_{l1}v & (x\in[0,s(t)], t\in[0,T], \mathbf{U}\in\mathbb{R}^2) \\ f_{l2}(\mathbf{U}) = r_{l2} - \beta_{l2}u - \gamma_{l2}v & (x\in(s(t),L], t\in[0,T], \mathbf{U}\in\mathbb{R}^2), \end{cases}$$

and where r_{li} are positive constants, and β_{li}, γ_{li} are nonnegative constants. We see that the coefficients of the equations in (1.1) and (1.2) are allowed to be discontinuous on curve $x = s(t)$.