Nonlinear Analysis: Real World Applications 45 (2019) 186–199

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

An integro-PDE model with variable motility

Bang-Sheng Han[a,b], Yinghui Yang[a,*]

[a] School of Mathematics, Southwest Jiaotong University, Chengdu, Sichuan, 611756, People's Republic of China

[b] School of Civil Engineering, Southwest Jiaotong University, Chengdu, Sichuan, 611756, People's Republic of China

ARTICLE INFO

Article history:
Received 11 December 2017
Received in revised form 3 July 2018
Accepted 6 July 2018

Keywords:
Nonlocal
Reaction–diffusion equation
Mutation
Mathematical ecology

ABSTRACT

This paper is concerned with a nonlocal reaction–diffusion–mutation model. It involves the spatial variable and a trait variable which govern the spatial diffusion of species. By establishing comparison principle and constructing monotone iterative sequence, we have proved the existence of solution to Cauchy problem. Then, based on the quasi-elementary solution, auxiliary equation and method of successive improvement of upper and lower solutions, the solutions are shown to be unique, bounded and globally stable.

1. Introduction

In this paper, we study the following nonlocal reaction–diffusion–mutation model

$$\begin{cases} u_t = \theta u_{xx} + du_{\theta\theta} + u\Big\{1 + \alpha u - \beta u^2 - (1+\alpha-\beta) \\ \qquad \times \displaystyle\int_{\mathbb{R}}\int_{\Theta} k(x-y,\theta-\theta')u(y,\theta',t)d\theta' dy\Big\}, & (x,\theta,t) \in \mathbb{R}\times\Theta\times(0,+\infty), \\ u(x,\theta,0) = u_0(x,\theta), & (x,\theta)\in\mathbb{R}\times\Theta, \\ \dfrac{\partial u}{\partial\theta}(x,\theta,t) = 0, & (x,\theta,t)\in\mathbb{R}\times\partial\Theta\times(0,+\infty), \end{cases} \tag{1.1}$$

where $d > 0$, $\alpha > 0$, $0 < \beta < 1 + \alpha$ and k, u_0 are nonnegative continuous functions. In addition, $\Theta := (\theta_{min}, \theta_{max})$ is a bounded subset of $(0, \infty)$.

The motivation of our focusing on (1.1) comes from modeling the dynamics of population (especially the cane toad population) subject to selection, mutation and migration, where the phenotypical trait (for instance size of legs) θ influences the dispersion rate. The pioneering work of this problem could be originated

* Corresponding author.
E-mail address: yangyh8605@swjtu.edu.cn (Y. Yang).

https://doi.org/10.1016/j.nonrwa.2018.07.004

from Phillips et al. [1]. They studied the invasion of the cane toads in Australia and found that the toads have strong ability of mutation. This could be reflected on the fact that toads arriving first in the new areas have longer legs. In addition, they also found the spreading speed of the toads is increasing with time. Because it was differently illustrated by the results of the classical Fisher–KPP equation, then Bénichou et al. [2] modified the classical Fisher–KPP equation and gave the following equation to describe the phenomenon in [1],

$$u_t = \theta u_{xx} + du_{\theta\theta} + ru\left(1 - \int_{\Theta} u(x,\theta,t)d\theta\right) \quad \text{for } (x,\theta,t) \in \mathbb{R} \times \Theta \times (0,\infty). \tag{1.2}$$

However, they ignored the advantage of the toads clustering and did not refine the competition (like dividing into resources competition and space competition). In order to make the model more realistic, inspired by [3–10], we extended Eq. (1.2) to the new one (1.1). In (1.1), x is the space variable. θ represents motility traits and parameter $d > 0$ means the rate of mutation. In addition, αu term can indicate the advantage of individuals when locally aggregated. Competition processes for food resources and space are reflected by the integral term and $-\beta u^2$.

Recently, such type of model is gaining a lot of attentions from mathematical biology community. And several works have been done on the variants of Eq. (1.1). Alfaro et al. [11] once studied

$$u_t - \triangle_{x,\theta} u = \left(r(\theta - x) - \int_{\mathbb{R}} k(\theta - x, \theta' - x)u(x,\theta',t)d\theta'\right)u, \tag{1.3}$$

where the trait only affects the growth rate of species. They proved that Eq. (1.3) exists traveling wave solutions for speeds beyond a critical threshold. Berestycki et al. [12] concerned a similar equation and showed the existence, uniqueness of the traveling wave solution for the corresponding equation. Further, they gave the asymptotic speed of propagation. Simultaneously, by focusing on the traveling wave solution of Eq. (1.2), Bouin and Calvez [13] also proved that (1.2) exists traveling wave solution when $c = c^*$, where c^* is the minimal speed. The study on the asymptotic spreading of solutions to cane toads equations started with a Hamilton–Jacobi framework which was formally developed in [14,15]. And the framework was rigorously justified in [16] when θ belongs to a finite interval. More recently, Lam and Lou [17] studied the asymptotic profile of the positive steady state solutions when the mutation rate is sufficiently small. They showed that the solution remains regular in the spatial variables and yet concentrates in the trait variable and forms a Dirac mass supported at the lowest diffusion rate. Furthermore, by means of the corresponding nonlocal eigenvalue problem and degree argument, Lam [18] proved the stability and uniqueness of the positive steady state solutions. In addition, Bouin et al. [19] focused on the super-linear propagation in (1.2) and proved the population spreads as $t^{3/2}$ when $\Theta = (\underline{\theta}, +\infty)$. Some other researches about the mutation–selection models with trait can be seen [20–23] and their references.

Most of these above works were only focusing on the qualitative behavior of the dynamics, including traveling wave solution, the asymptotic speed of propagation and so on. However, the well-posedness of solutions for the Cauchy problem was rarely referred. It is also a very important question. Our paper here will study the behavior of solution from the Cauchy problem (1.1), including the existence, uniqueness and global stability. Compared with the model without mutation and trait, (1.1) not only includes a space variable x, but also involves a trait θ, being with one more variables, which hence leads to more complication. It is worth noting that the trait θ affects the spread of diffusion, which means (1.1) is a variable coefficients equation. This makes it harder to obtain the elementary solution for the model (1.1). In addition, similar to [6,10,24–26], there is no maximum principle since the reaction term contains integral term. To overcome these difficulties, we establish the quasi-elementary solution and new comparison principle for the Cauchy problem (1.1).

The paper is organized as follows. In Section 2, we define a pair of upper and lower solutions of (1.1) and establish the comparison principle. By constructing two monotone sequences of upper and low solutions, we prove the existence of the solutions from Cauchy problem (1.1). In Section 3, the solution is shown to be bounded, unique and globally stable.