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Boundedness in chemotaxis–Stokes system with rotational flux term[☆]

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ABSTRACT

We consider the following chemotaxis–Stokes system with rotation

$$\begin{cases} n_t = \Delta n - \nabla\cdot(nS(x,n,c)\cdot\nabla c) - u\cdot\nabla n, \\ c_t = \Delta c - f(x,n,c) - u\cdot\nabla c, \\ u_t = \Delta u + \nabla P + n\nabla\phi, \\ \nabla\cdot u = 0 \end{cases}$$

in $\Omega\times(0,T)$, subject to the non-flux boundary conditions for n and$c$, as well as the Dirichlet boundary condition for u, where the bounded smooth domain $\Omega\subset\mathbb{R}^3$, the matrix-valued function $S\in C^2(\bar\Omega\times[0,\infty)^2;\mathbb{R}^{3\times3})$ fulfills $|S(x,n,c)|\le\frac{S_0(c)}{(1+n)^\theta}$ for all $(x,n,c)\in\bar\Omega\times[0,\infty)^2$ with S_0 nondecreasing, and $f\in C^1(\bar\Omega\times[0,\infty)^2;\mathbb{R})$ satisfies $0\le f(x,n,c)\le f_0(c)(n+1)$ with f_0 nondecreasing and $f(x,n,0)=0$. It was proved that for any $\theta>0$, the initial–boundary value problem possesses a unique globally bounded classical solution.

1. Introduction

In this paper, we consider the chemotaxis–Stokes system

$$\begin{cases} n_t = \Delta n - \nabla\cdot(nS(x,n,c)\cdot\nabla c) - u\cdot\nabla n, & (x,t)\in\Omega\times(0,T), \\ c_t = \Delta c - f(x,n,c) - u\cdot\nabla c, & (x,t)\in\Omega\times(0,T), \\ u_t = \Delta u + \nabla P + n\nabla\phi, & (x,t)\in\Omega\times(0,T), \\ \nabla\cdot u = 0, & (x,t)\in\Omega\times(0,T), \\ \nabla c\cdot\nu = (\nabla n - S(x,n,c)\nabla c)\cdot\nu = 0, u = 0, & (x,t)\in\partial\Omega\times(0,T), \\ n(x,0) = n_0(x),\ c(x,0) = c_0(x),\ u(x,0) = u_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

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where $\Omega \subset \mathbb{R}^3$ is a bounded domain with smooth boundary $\partial\Omega$, ν denotes the outward normal vector on $\partial\Omega$, and $\phi \in W^{2,\infty}(\Omega)$. Here we assume that $S \in C^2(\bar{\Omega} \times [0,\infty)^2; \mathbb{R}^{3\times 3})$ satisfies

$$|S(x,n,c)| \le \frac{S_0(c)}{(1+n)^\theta} \quad \text{for all } (x,n,c) \in \bar{\Omega} \times [0,\infty)^2 \tag{1.2}$$

with non-decreasing function $S_0 : [0,\infty) \mapsto \mathbb{R}$ and $\theta > 0$, and that the nonnegative function $f \in C^1(\bar{\Omega} \times [0,\infty)^2; \mathbb{R})$ fulfills

$$f(x,n,0) = 0 \quad \text{for all } (x,n) \in \bar{\Omega} \times [0,\infty) \tag{1.3}$$

and

$$f(x,n,c) \le f_0(c)(n+1) \quad \text{for all } (x,n,c) \in \bar{\Omega} \times [0,\infty)^2 \tag{1.4}$$

with non-decreasing function $f_0 : [0,\infty) \mapsto \mathbb{R}$. The model (1.1) was proposed by Tuval [1] to describe the motion of oxygen-driven swimming cells in an incompressible fluid, where n and c denote the density of bacteria and the concentration of oxygen, respectively, and u represents the velocity field of the fluid subject to an incompressible Stokes equation with pressure P and gravitational force ϕ. This mechanism is an important variation of the chemotaxis model, which has been extensively studied in the past 40 years. Refer to the surveys [2–4] for a broad view.

A special case $S = \chi \cdot \mathbb{I}$ with $\chi \in \mathbb{R}$ was considered in [1], where the bacteria always move toward the higher concentration of oxygen. Correspondingly, the chemotaxis–(Navier)–Stokes model reads as

$$\begin{cases} n_t = \Delta n - \nabla \cdot (\chi(c) n \nabla c) - u \cdot \nabla n, \\ c_t = \Delta c - n f(c) - u \cdot \nabla c, \\ u_t = \Delta u - \kappa (u \cdot \nabla) u + \nabla P + n \nabla \phi, \\ \nabla \cdot u = 0. \end{cases} \tag{1.5}$$

Lots of results for (1.5) have been obtained via a natural gradient-like functional with

$$\frac{d}{dt}\Big(\int_\Omega n \ln n + \frac{1}{2}\int_\Omega \frac{|\nabla c|^2}{c}\Big) + \int_\Omega \Big(\frac{|\nabla n|^2}{n} + c|D^2 \ln c|^2\Big) \le C \int_\Omega |u|^4. \tag{1.6}$$

See, e.g., [5–12] and the references therein.

It was mentioned that a wide variety of S has been suggested in [13] due to complicated interaction environments around cells. When the cells or organisms move with a rotating motion, rather than directed toward the concentration of oxygen, the chemotactic sensitivity should be a tensor. This leads to the system

$$\begin{cases} n_t = \Delta n - \nabla \cdot (n S(x,n,c) \cdot \nabla c) - u \cdot \nabla n, \\ c_t = \Delta c - f(x,n,c) - u \cdot \nabla c, \\ u_t = \Delta u - \kappa (u \cdot \nabla) u + \nabla P + n \nabla \phi, \\ \nabla \cdot u = 0. \end{cases} \tag{1.7}$$

For the fluid-free case (that is $u = 0$) with S satisfying (1.2), $\theta = 0$, and $\|c_0\|_{L^\infty(\Omega)}$ sufficiently small, it was shown in [14] that the initial–boundary value problem of (1.7) admits global classical solutions in two dimensional bounded convex domains. The same problem for large initial data was studied in [15] which showed that there exists at least one global very weak generalized solution which converges to the constant steady state in the large time limit. Recently, it was extended in [16] that the above problem possesses a global classical solution under one of the following conditions: (i) $N = 1$; (ii) $N \ge 2, \theta = 0$ and $S_0(\|c_0\|_{L^\infty(\Omega)})\|c_0\|_{L^\infty(\Omega)} \le \frac{2}{\sqrt{3N(11N+2)}}$; (iii) $\theta > 0$. In addition, an explicit upper bound requirement to $\|c_0\|_{L^\infty(\Omega)}$ (dependent only on S_0 and N) was obtained to ensure the boundedness of classical solutions.