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Existence and continuity of solution trajectories of generalized equations with application in electronics

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ABSTRACT

We consider a special form of parametric generalized equations arising from electronic circuits with AC sources and study the effect of perturbing the input signal on solution trajectories. Using methods of variational analysis and strong metric regularity property of an auxiliary map, we are able to prove the regularity properties of the solution trajectories inherited by the input signal. Furthermore, we establish the existence of continuous solution trajectories for the perturbed problem. This can be achieved via a result of uniform strong metric regularity for the auxiliary map.

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1. Introduction

This paper deals with *parametric generalized equations* of the form

$$0 \in f(x) - p(t) + F(x), \tag{1}$$

where $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a (usually) smooth function, $p : \mathbb{R} \longrightarrow \mathbb{R}^n$ is a function of parameter $t \in [0,1]$, and $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is a set-valued map with closed graph.

Generalized equations (when p is a constant function) has been well studied in the literature of variational analysis (see, for instance, [1,2] and references therein). Robinson in [3–6] studied in detail the case where F is the normal cone at a point $x \in \mathbb{R}^n$ (in the sense of convex analysis) to a closed and convex set, and found the setting of generalized equations as an appropriate way to express and analyze problems in complementarity systems, mathematical programming, and variational inequalities.

In recent years, the formalism of generalized equations has been used to describe the behavior of electronic circuits [7–9]. In the study of electrical circuits, power supplies (that is, both current and voltage sources)

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play an important role. Not only their failure in providing the minimum voltage level for other components to work would be a problem, but also small changes in the provided voltage level will affect the whole circuit and the goal it has been designed for. These small changes around a desired value could happen mainly because of failure in precise measurements, ageing process, and thermal effects (see [10,11]). Especially, electronic components are very sensitive to thermal changes and demanding a way to analyze and design high precision tools has been a main concern in instrumentation engineering. The widespread usage of electronic devices in any aspect of our daily lives with an increasing demand for more precision in these tools has created the need for analyzing sensitivity not only as part of the instrumentation engineering but in design level of any electronic product. This paper aims to provide a new mathematical prospective for this analysis.

Based on the type of voltage/current sources in the circuits, two different cases may be considered:

1. **static case:**
   This is the situation when the signal sources in the circuit are DC (that is, their value is not changing with respect to time). For practical reasons, we would prefer to rewrite (1) as $p \in f(x) + F(x)$, where $p \in \mathbb{R}^n$ is a fixed vector representing the voltage or current sources in the circuit, and $x \in \mathbb{R}^n$ represents the mixture of n variables which are unknown currents of branches or voltages of components. The corresponding *solution mapping* can be defined as follows:
   $$p \longmapsto S(p) := \{x \in \mathbb{R}^n \mid p \in f(x) + F(x)\}. \tag{2}$$
   In this framework, small deviations of x with respect to perturbations of p around a presumed point $(\bar{p}, \bar{x}) \in \operatorname{gph} S$ could be formulated in terms of local stability properties of S at $\bar{p}$ for $\bar{x}$. Providing some first order and second order criteria to check the local stability properties of a set-valued map has been the subject of many papers [2,7,8,12] to mention a few.
2. **time-varying case:**
   When an AC signal source (that is, its value is a function of time) is in the circuit, the problem could be more complicated. First of all, the other variables of the model would become a function of time, too. Second, it is not appropriate any more to formulate the solution mapping as $S(p)$. One can consider a parametric generalized equation like (1) where p now depends on a scalar parameter $t \in [0, 1]$,[1] and define the corresponding solution mapping as
   $$S : t \mapsto S(t) = \{x \in \mathbb{R}^n \mid -p(t) + f(x) + F(x) \ni 0\}. \tag{3}$$
   The third difficulty rises here: the study of the effects of perturbations of p is not equivalent any more to searching the local stability properties of S.
   Clearly, in this framework for any fixed $t \in [0, 1]$ one has a static case problem. This approach is well known and well studied in the literature, both as a pointwise study (see for instance [2,13–15] and references therein), or as a numerical method and for designing algorithms (see for example [16–20]). In this paper, instead of looking at the sets $S(t)$, we focus on *solution trajectories*, functions like $x : [0, 1] \to \mathbb{R}^n$ such that $x(t) \in S(t)$, for all $t \in [0, 1]$, that is, $x(\cdot)$ is a selection for S over $[0, 1]$.

The main aim of this paper is to investigate the dependence of the solution trajectories on the regularity and changes of the input signal p. A side goal is to develop a relationship between pointwise stability properties and their "uniform" version. This has been done with reference to the property of strong metric regularity. It is worthwhile noting that, recently, a result very close in spirit has been obtained by Cibulka et al. in [21] as a byproduct, while investigating a differential generalized equation.

[1] In fact, t can belong to any finite interval like $[0, T]$ for a $T > 0$. The starting point $t = 0$ is considered as the moment that the circuit starts working, in other words, when the circuit is connected to the signal sources and is turned on with a key. We keep the time interval as $[0, 1]$ in the entire paper for simplicity.

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