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Global dynamics for an attraction–repulsion chemotaxis–(Navier)–Stokes system with logistic source

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ABSTRACT

This paper deals with an attraction–repulsion chemotaxis–(Navier)–Stokes model with logistic source

$$\begin{cases} n_t + \mathbf{u}\cdot\nabla n = \Delta n - \chi\nabla\cdot(n\nabla c) & \\ \quad +\xi\nabla\cdot(n\nabla v) + \mu n(1-n), & (x,t)\in\Omega\times(0,\infty),\\ c_t + \mathbf{u}\cdot\nabla c = \Delta c - c + n, & (x,t)\in\Omega\times(0,\infty),\\ v_t + \mathbf{u}\cdot\nabla v = \Delta v - v + n, & (x,t)\in\Omega\times(0,\infty),\\ \mathbf{u}_t + \kappa(\mathbf{u}\cdot\nabla)\mathbf{u} = \Delta\mathbf{u} + \nabla P + n\nabla\phi, & (x,t)\in\Omega\times(0,\infty),\\ \nabla\cdot\mathbf{u} = 0, & (x,t)\in\Omega\times(0,\infty), \end{cases}$$

under homogeneous Neumann boundary conditions in a smooth bounded domain $\Omega\subset\mathbb{R}^N$, $N=2,3$, where $\kappa\in\{0,1\}$, the parameters χ, ξ andμ are positive. This system describes the evolution of cells which react on two different chemical signals in a liquid surrounding environment. The cells and chemical substances are transported by a viscous incompressible fluid under the influence of a force due to the aggregation of cells. Firstly, when $N=2$ and $\kappa=1$, based on the standard heat-semigroup argument, it is proved that for all appropriately regular nonnegative initial data and any positive parameters, this system possesses a unique global bounded solution. Secondly, when $N=3$ and $\kappa=0$, by using the maximal Sobolev regularity and semigroup technique, it is proved that the system admits a unique globally bounded classical solution provided that there exists $\theta_0>0$ such that $\frac{\chi+\xi}{\mu}<\theta_0$. Finally, by means of energy functionals, it is shown that the global bounded solution of the above system converges to the constant steady state. Furthermore, we give the precise convergence rates.

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1. Introduction

In this paper, we consider the following attraction–repulsion chemotaxis model under the effect of an incompressible viscous fluid with logistic source

$$\begin{cases} n_t + \mathbf{u}\cdot\nabla n = \Delta n - \chi\nabla\cdot(n\nabla c) + \xi\nabla\cdot(n\nabla v) + \mu n(1-n), & (x,t)\in\Omega\times(0,\infty), \\ c_t + \mathbf{u}\cdot\nabla c = \Delta c - c + n, & (x,t)\in\Omega\times(0,\infty), \\ v_t + \mathbf{u}\cdot\nabla v = \Delta v - v + n, & (x,t)\in\Omega\times(0,\infty), \\ \mathbf{u}_t + \kappa(\mathbf{u}\cdot\nabla)\mathbf{u} = \Delta\mathbf{u} + \nabla P + n\nabla\phi, & (x,t)\in\Omega\times(0,\infty), \\ \nabla\cdot\mathbf{u} = 0, & (x,t)\in\Omega\times(0,\infty), \\ \dfrac{\partial n}{\partial\nu} = \dfrac{\partial c}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, \quad \mathbf{u} = \mathbf{0}, & (x,t)\in\partial\Omega\times(0,\infty), \\ n(x,0) = n_0(x), c(x,0) = c_0(x), v(x,0) = v_0(x), \mathbf{u}(x,0) = \mathbf{u}_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where $\kappa\in\{0,1\}$, $\chi>0$, $\xi>0$, $\mu>0$, $\Omega\subset\mathbb{R}^N$ $(N=2,3)$ is a bounded domain with smooth boundary $\partial\Omega$, $\frac{\partial}{\partial\nu}$ denotes the differentiation with respect to the outward normal of $\partial\Omega$. The functions $n_0, c_0, v_0, \mathbf{u}_0$ and ϕ are well-known and satisfy

$$0 < n_0 \in C(\overline{\Omega}), \quad 0 < c_0, v_0 \in W^{1,q}(\Omega), \quad \mathbf{u}_0 \in D(A^\theta), \quad \phi\in C^{1+\eta}(\overline{\Omega}) \tag{1.2}$$

for some $q>N$, $\theta\in(\frac{N}{4},1)$, $\eta>0$ and A is the Stokes operator. Here $n=n(x,t)$, $c=c(x,t)$, $v=v(x,t)$, $P(x,t)$ and $\mathbf{u}(x,t)$ denote respectively the cell density, the concentration of an attractive chemical signal, the concentration of a repulsive chemical signal, the hydrostatic pressure, and the velocity field of the fluid at position $x\in\Omega$ and time $t\in(0,\infty)$. System (1.1) consists of three extensions of the classical Keller–Segel model [1]. The first one is the chemotaxis–(Navier)–Stokes model, which is known as a challenging model; the second extension corresponds to the attractive–repulsive chemotaxis frameworks, and the third one corresponds to the chemotaxis models with logistic source. However, from the mathematical point of view, system (1.1) seems not to have been studied yet.

In order to understand the development of system (1.1), let us mention some previous contributions in this direction. In recent years, the following attraction–repulsion chemotaxis model without fluid and logistic source

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w), & (x,t)\in\Omega\times(0,\infty), \\ \tau v_t = \Delta v + \alpha u - \beta v, & (x,t)\in\Omega\times(0,\infty), \\ \rho w_t = \Delta w + \gamma u - \delta w, & (x,t)\in\Omega\times(0,\infty), \end{cases} \tag{1.3}$$

was studied by several authors [2–8], where $\tau,\rho\in\{0,1\}$ and the parameters $\chi,\xi,\alpha,\beta,\gamma$ and δ are positive. In particular, Tao and Wang [7] considered the global solvability, boundedness, blow-up, existence of steady states, and asymptotic behavior of system (1.3) for various ranges of parameter values. Particularly, it has been shown that if $\chi\alpha-\xi\gamma<0$, system (1.3) with $\tau=\rho=0$ is globally well-posed in high dimensions $n\geq 2$, while $\chi\alpha-\xi\gamma<0$ and $\beta=\delta$, system (1.3) with $\tau=\rho=1$ is globally well-posed in two dimensions. Liu and Wang [5] studied the global existence, asymptotic behavior and steady states of solutions to system (1.3) with $\tau=\rho=1$ in one dimension. Jin [4] proved the global existence of classical solution in two dimensions and weak solution in three dimensions with large initial data provided that $\chi\alpha-\xi\gamma<0$ in (1.2). Moreover, when $\tau=1$ and $\rho=0$ in (1.3), Jin and Wang [9] investigated boundedness, blowup and critical mass phenomenon for (1.3).

Furthermore, the attraction–repulsion chemotaxis model with logistic source but without fluid

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + f(u), & (x,t)\in\Omega\times(0,\infty), \\ \tau v_t = \Delta v + \alpha u - \beta v, & (x,t)\in\Omega\times(0,\infty), \\ \rho w_t = \Delta w + \gamma u - \delta w, & (x,t)\in\Omega\times(0,\infty), \end{cases} \tag{1.4}$$