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Boundary control of small solutions to fluid–structure interactions arising in coupling of elasticity with Navier–Stokes equation under mixed boundary conditions

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ABSTRACT

We consider a coupled system of a linearly elastic body immersed in a flowing fluid which is modeled by means of the incompressible Navier–Stokes equations with mixed Dirichlet–Neumann-type boundary conditions. For this system we formulate an optimal control problem which amounts to a minimization under constraints of a hydro-elastic pressure at the interface between the two environments. The corresponding functional lacks convexity and radial unboundedness — a serious obstacle to the solution of the optimization problem. The approach taken to solve it is based on transforming the variable domain occupied by the fluid into a fixed domain corresponding to the undeformed elastic inclusion. This leads to a free boundary elliptic problem. Mathematical challenge also results from the fact that the corresponding quasilinear elliptic model is equipped with mixed (Zaremba type) boundary conditions, which intrinsically lead to compromised regularity of elliptic solutions. It is shown that under the assumption of small strains, the controlled structure is wellposed in suitable Sobolev's spaces and the nonlinear control-to-state map is well defined and continuous. The obtained wellposedness result provides thus foundation for proving existence of an optimal control, where the latter is based on compensated compactness methods. The change in the boundary conditions makes the analysis different and substantially more challenging — particularly at the level of wellposedness of both uncontrolled and controlled dynamics. A key to the existence/uniqueness theory is a suitable localization of the spatial domain and of the resulting PDE. The geometry of the fluid domain plays a critical role in the arguments. The analysis of optimal control is nonstandard due to the lack of convexity and of radial unboundedness of the associated functional cost — main tools for proving weak lower-semicontinuity of the functional. This difficulty is dealt with the help of compensated compactness.

1. Introduction

Mathematical models for fluid–structure interactions (FSI) have attracted considerable attention in the literature. This is due to a broad array of applications ranging from fluid dynamics and aeroelasticity

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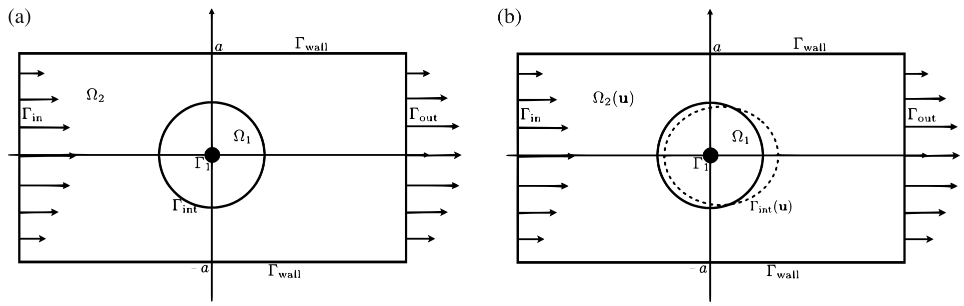

Fig. 1. Domain $D = \Omega_1 \cup \Omega_2$ with its boundary $\Gamma_{\text{in}} \cup \Gamma_{\text{out}} \cup \Gamma_{\text{wall}}$.

to biological, life sciences and medicine. Specific examples are abundant. Problems such as interaction of gas/fluid flow surrounding the structure (submarine submersed in water, wing of an airplane in a viscous gas), or fluid inside a structure (body fluids in the cardiovascular systems) are prime examples of applications modeled by FSI [1–7]. These models involve coupled PDEs consisting of a fluid equation (typically NS equation) and a structure equation (typically elastic body, plate or shell). The pivotal role in the analysis is played by the interface between the two media, which by itself is unknown and is determined by deformations of the structure. Thus, we deal with a "free" boundary problem. External boundary conditions depend on the type of application under consideration. A mathematical theory of such models is still under development and its complexity and challenge depend on (i) the type of boundary data prescribed and (ii) the geometry of the domain. Since the overall mathematical model leads to a quasilinear system, *regularity* of the corresponding elliptic solutions plays a dominant role. It is well known that *regularity* of elliptic solutions with regular input data can be *compromised* by two factors: (i) singularity of the domain [8], (ii) mixed type of boundary conditions referred often to as Zaremba problem [9]. On the other hand, specific applications call for such domains/scenarios to be considered [10]. While FSI, in both static [11,12] and dynamic [13–17] forms, have been treated, these works refer to "regular" configurations with respect to the domain and homogeneous boundary conditions. What distinguishes our work are two facts: (i) that we deal with "corners" and mixed Dirichlet/Neumann boundary conditions imposed on the same part of the boundary, (ii) that we treat boundary control problem with Dirichlet non-homogeneous data, with the goal of reducing the aerodynamic resistance due to build-up of the pressure. The change in the boundary conditions makes the analysis different and substantially more challenging. In addition, a control problem associated with the FSI is also studied. In what follows we shall describe the problem studied. The analysis in the stationary case is a first step toward extending the results to dynamic models where the treatment of both local and global (in time) solutions has been considered in the case of *smooth boundaries and with zero Dirichlet data* imposed on the external boundary of the fluid [15,18,19].

2. Problem formulation

Let $D \subset \mathbb{R}^n$, $n = 2, 3$ be a bounded domain with a piecewise regular boundary ∂D consisting of two sub-domains Ω_1 and Ω_2, as shown in Fig. 1(a). The boundary $\partial\Omega_1$ of the interior doughnut-like domain Ω_1 is denoted by $\Gamma_{\text{int}} \cup \Gamma_1$ while the exterior boundary $\partial\Omega_2$ is denoted by $\Gamma_{\text{ext}} = \Gamma_{\text{in}} \cup \Gamma_{\text{out}} \cup \Gamma_{\text{wall}}$. In the interior subdomain Ω_1 we consider the problem of linear elasticity for an elastic body with $\mathbf{u}$ denoting the displacement field. In the exterior subdomain Ω_2 we consider a Navier–Stokes problem for the motion of a fluid with $\tilde{\mathbf{w}}$ denoting the velocity field. As will be seen below, the resulting system is quasilinear with a free boundary. The two goals of the paper are: (a) to provide a wellposedness theory (existence, uniqueness and continuous dependence on the data) for the corresponding solution, and (b) to establish existence of a