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Transition waves for two species competition system in time heterogeneous media

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A B S T R A C T

This paper is concerned with the spreading speeds and transition waves of the following two species competition system in time heterogeneous media,

$$\begin{cases} u_t = \Delta u + u(a_1(t) - b_1(t)u - c_1(t)v), \\ v_t = \Delta v + v(a_2(t) - b_2(t)u - c_2(t)v), \end{cases} \quad x \in \mathbb{R}^N, t \in \mathbb{R},$$

where $a_i(\cdot), b_i(\cdot), c_i(\cdot)$ $(i = 1, 2)$ depend in a general way in $t \in \mathbb{R}$. The notion of transition waves for such a system is first introduced in this paper. We first establish the lower bounds of spreading speed intervals and generalized spreading speed intervals. It then shows that, under certain conditions, there exists a pair of general transition waves $(u(t, x), v(t, x))$ connecting two semitrivial solutions of this system for a given class of speed $c(t)$ with $\underline{c} > c^*$ and for any $\underline{c} < c^*$, there is no such transition waves.

1. Introduction

This paper is concerned with the spatial spreading speeds and transition waves of the following two species competition system in time heterogeneous media,

$$\begin{cases} u_t = \Delta u + u(a_1(t) - b_1(t)u - c_1(t)v), \\ v_t = \Delta v + v(a_2(t) - b_2(t)u - c_2(t)v), \end{cases} \quad x \in \mathbb{R}^N, t \in \mathbb{R}, \tag{1.1}$$

where $u(t, x)$ and $v(t, x)$ represent the population densities of two species, and $a_i(\cdot), b_i(\cdot)$, $c_i(\cdot)$ $(i = 1, 2)$ are $L^\infty(\mathbb{R})$ in $t \in \mathbb{R}$, $b_i(t) > 0$ and $c_i(t) > 0$ for $t \in \mathbb{R}$. System (1.1) usually describes the population dynamics of two competing species with internal interaction in time heterogeneous environment and the coefficients $a_i(\cdot), b_i(\cdot), c_i(\cdot)$ $(i = 1, 2)$ depend in a general way in $t \in \mathbb{R}$, which reflect time heterogeneous of the environment.

There have been many interesting works on spreading speed and traveling wave solution of two species competition system with both time independent and time periodic case. For example, see [1–21] etc. Recently,

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the study of spatial spreading speeds and transition waves of reaction–diffusion equation with general time and/or space dependence have attracted a lot of attention. The notion of generalized transition waves for general heterogeneous reaction–diffusion equations was introduced by Berestycki and Hamel [22,23] and Shen [24], respectively. Shen [25] proved the existence, uniqueness and stability of generalized traveling waves for spatially homogeneous monostable reaction–diffusion equation with ergodic or recurrent time dependence, which includes periodic and almost periodic time dependence as special cases. Shen [26] established the existence of generalized transition wave for time heterogeneous bistable reaction–diffusion equation. Nadin and Rossi [27] proved the existence of generalized transition waves of reaction–diffusion equation with general time dependent KPP nonlinearity. They also applied their results to random stationary ergodic equations and show the existence of random transition waves when nonlinear term is a random stationary ergodic function with respect to t. Rossi and Ryzhik [28] proved the existence of generalized transition waves for KPP equation depending explicitly on time and space and a lower bound for speeds is obtained for KPP equation with non-periodic spatially dependence coefficients. Hamel and Rossi [29] studied the set of admissible asymptotic past and future speeds of transition fronts for KPP equation depending on $t \in \mathbb{R}$. Nadin and Rossi [30] also proved the existence and non-existence of generalized transition waves of KPP equations with time heterogeneous and space-periodic coefficients. Shen and Shen [31] proved the existence, uniqueness and stability of generalized transition waves of nonlocal Fisher–KPP equation with time dependent. Cao and Shen [32] studied the spreading speeds and transition fronts of discrete fisher KPP equation in time heterogeneous. For more results on transition waves of reaction–diffusion equations, we refer to [24,28,33–43] and so on.

Although the study on traveling wave solutions and spreading speeds of two species competition system with autonomous and periodic case have a longstanding history, there are still very few studies on spatial spreading and transition waves for diffusive systems with time non-periodic reaction terms. It is worth to point out that Bao et al. [44] have introduced the notion of spreading speeds for general time dependents cooperative system with different dispersal types, including (1.1). In the current paper, we will extend the notion of generalized transition waves introduced in [22–24] for scalar reaction–diffusion equation to two species competition system (1.1) and try to study the spatial spreading speed and transition waves for (1.1) with time general dependent.

The corresponding ordinary differential system of (1.1) is

$$\begin{cases} \dfrac{du}{dt}(t) = u(a_1(t) - b_1(t)u - c_1(t)v), \\ \dfrac{dv}{dt}(t) = v(a_2(t) - b_2(t)u - c_2(t)v), \end{cases} \qquad t \in \mathbb{R}. \tag{1.2}$$

Note that $(0,0)$ is the trivial solution of (1.2). Throughout this paper, we assume that the trivial solution $(0,0)$ is completely unstable with respect to perturbation in $\mathbb{R}^+ \times \mathbb{R}^+$, that is

(H1) $\lim_{T\to\infty}\inf_{t\in\mathbb{R}}\frac{1}{T}\int_t^{t+T} a_i(s)ds > 0$, $i = 1,2$.

We will use the notation $\underline{a} := \lim_{T\to\infty}\inf_{t\in\mathbb{R}}\frac{1}{T}\int_t^{t+T} a(s)ds$ for the least mean of the function $a(t)$. In fact, (H1) implies that (1.2) has two semitrivial positive solutions $(u^*(t),0)$ and $(0,v^*(t))$ in $\mathbb{R}^+ \times \mathbb{R}^+$, (see Proposition 2.1). We also assume that

(H2) $(0,v^*(t))$ is linearly unstable in $\mathbb{R}^+ \times \mathbb{R}^+$ and $(u^*(t),0)$ is linearly and globally stable in $\mathbb{R}^+ \times \mathbb{R}^+$.

The assumption (H2) implies that the species u can completely invade the species v and the species v cannot invade the species u. We remark that (H2) holds if the coefficients $a_i(t)$, $b_i(t)$ and $c_i(t)\,(i=1,2)$ satisfying $a_{1L} > \frac{c_{1M}a_{2M}}{c_{2L}}$ and $a_{2M} \le \frac{a_{1L}b_{2L}}{b_{1M}}$, where $a_{iL} = \inf_{t\in\mathbb{R}}a_i(t)$, $a_{iM} = \sup_{t\in\mathbb{R}}a_i(t)$, and $b_{iL}, b_{iM}, c_{iL}, c_{iM}$ are defined similarly (see Proposition 2.4).