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Global dynamics in sea lice model with stage structure[☆]

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ABSTRACT

Sea lice infection is one of the major threats in the marine fishery, especially for farmed salmon. In this paper, we propose a mathematical model for the growth of sea lice with a three-stage structure: non-infectious immature, infectious immature and adults where the level of non-infectious immature development depends on the size of the adult population. We first describe the nonlinear dynamics by a system of partial differential equations, then, by mathematical techniques and an appropriate change of variables transform it into a system of delay differential equations with constant delay. We address the system threshold dynamics in the established model with respect to the adult reproduction number $\mathcal{R}_s$, including the global stability of the trivial steady state when $\mathcal{R}_s < 1$, persistence and global attractivity of the unique positive steady state when $\mathcal{R}_s > 1$. Numerical simulations are provided to confirm the theoretical results.

1. Introduction

Since the 1970s, salmon production in farms has increased exponentially throughout Canada, Chile, Ireland, Norway and Scotland. The number of farmed salmon was growing from a few thousand tonnes in 1980 to about 2.5 million tonnes in 2014. Sea lice infection became one of the major threats on farmed salmon during the past 40 years [1,2]. Sea lice are marine ectoparasites feeding on the mucus and tissue of host marine fish. In salmon-producing countries, salmon lice are responsible for many outbreaks of disease in salmonid aquaculture, causing enormous economic losses in the salmon aquaculture industry and costing millions of dollars annually [2,3]. According to the Atlantic Salmon Federation, the costs of fighting sea lice is growing. For instance, in Norway, there was an increase of 32.5% in the cost of sea lice mitigation. Therefore, it is essential to develop mathematical models to predict the variations in sea lice abundance.

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After the age-structured population model designed by McKendrick in 1926 [4], there has been a fair amount of work on modeling populations with various stages in life history [5–18]. For example, the authors proposed a stage-structured model of single species with two life stages, immature and mature and with a constant time from birth to maturity in [9]. In [18], the authors formulated and analyzed a single-species growth model with stage-structure consisting of immature and mature stages for the effects of toxicants with constant maturation time delay. In [10], the author considered a prey-predation model with a maturation time in predator, studied the stability and bifurcation with or without the maturation time delay. In [6,11], parasite life stage models were derived with periodic delays due to the seasonally varying temperature on parasite maturation. The basic reproduction ratio is introduced and the long-term behavior of solutions is investigated in [11]. In a series of papers [13–16], Smith discussed a single species population model with two classes, immature and mature, where the immature individuals grow at a non-constant rate. For instance, in [13], a competition between adults and juveniles is considered and it is assumed that the maturation rate of the juveniles depends on the density of adults, in the sense that, as the adult population increases, the rate of maturation of juveniles decreases, which causes juveniles to remain in the juvenile stage longer and therefore be exposed to increased mortality; while the maturation rate of the larvae depends on the food density in which the maturation rate increases as the food density increases in [16]. Recently, in [12], the authors used Smith's techniques to derive nutrient-phytoplankton–zooplankton model where the maturation rate of the juvenile zooplankton (predator) depends on the quantity of phytoplankton (prey).

The sea louse exhibits several distinct life stages in two separate phases: the free-living phase and the parasitic phase. In general, during the life cycle of a sea louse, eggs hatch into a non-infectious free-living nauplius, after 2 to 14 days this nauplius moults into an infective copepodid and starts searching for a host. Once attached to a host, the nauplius feeds on the mucous and skin of the host, begins to develop into infectious larvae. In this manuscript, we assume the growth of sea lice through three stages: non-infectious larvae and infectious larvae in the free-living phase, adults in the parasitic phase, and assume that the development age for non-infectious larvae to develop into infectious larvae depends on the size of adult population size, in the sense that, a larger mature population is more favorable than a smaller one for facilitating development. Biologically, this is reasonable, since the growth of immature and matured sea lice depends on different levels of food resources. The main goal of this work is to model the growth process and study the dynamical behavior of sea lice population. As a starting point, we adopt the idea from Smith's work [13–15] to construct a system of partial differential equations (PDE) for describing the dynamics, then by using the technique of integration along characteristics, we reduce the system to a threshold delayed differential equations system (TDE), then by changing variables, we remove the state-dependent delay in (TDE) and transform it into a standard time-delayed differential equations system (DDE). Based on the proposed delay mathematical model DDE, we study the nonlinear dynamics in the system including the threshold dynamics with respect to the adult reproduction number $\mathcal{R}_s$ under biologically reasonable conditions.

The rest of the manuscript is organized as follows: in Section 2, we propose a PDE system to describe the evolution of sea lice, solve the PDE model to construct a system of threshold type delay TDE, and transform the obtained TDE system into a DDE with constant delay. In Section 3, we discuss the well-posedness property by verifying the non-negativity and boundedness of the solutions with reasonable initial data, calculate the adult reproduction number $\mathcal{R}_s$ and address the local stability of the trivial equilibrium point. In Section 4, we establish the threshold dynamics for the system in terms of $\mathcal{R}_s$ by proving the global stability of the trivial equilibrium point when $\mathcal{R}_s < 1$, sea lice persistence, existence and global attractivity of positive steady state when $\mathcal{R}_s > 1$, and discuss the sensitivity of $\mathcal{R}_s$ with respect to the related parameters. In Section 5, we present some numerical simulations using *Lepeophtheirus salmonis* growth as a case study. Finally, conclusion and remarks are drawn in Section 6.