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# Stability of composite wave for inflow problem on the planar magnetohydrodynamics

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ABSTRACT

In this paper, we investigate the large time behavior of the solutions to an initial–boundary value problem for the planar magnetohydrodynamics in a half line $\mathbb{R}_+ := (0,\infty)$. Inspired by the relationship between magnetohydrodynamics and Navier–Stokes, we can prove that the composite wave consisting of the subsonic BL-solution, the contact wave, and the rarefaction wave for the inflow problem on the planar magnetohydrodynamics is time-asymptotically stable. Meanwhile, we obtain the global existence of solutions based on the basic energy method by taking into account the complexity of composite wave.



## 1. Introduction

Magnetohydrodynamics (MHD) concerns the motion of a conducting fluid in an electro-magnetic field with a very wide range of applications in astrophysics, plasma, and so on. There is a complex interaction between the magnetic and fluid dynamic phenomena, and both hydrodynamic and electrodynamic effects have to be considered. The planar magnetohydrodynamics in the half line $\mathbb{R}_+ =: (0,+\infty)$ is governed by the following equations in Eulerian coordinates:

$$\begin{cases} \partial_t\rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2 + p + \frac{1}{2}|\mathbf{b}|^2) = \lambda\partial_x^2 u, \\ \partial_t(\rho\mathbf{w}) + \partial_x(\rho u\mathbf{w} - \mathbf{b}) = \mu\partial_x^2\mathbf{w}, \\ \partial_t\mathbf{b} + \partial_x(u\mathbf{b} - \mathbf{w}) = \nu\partial_x^2\mathbf{b}, \\ \partial_t\mathcal{E} + \partial_x\left(u(\mathcal{E} + p + \frac{1}{2}|\mathbf{b}|^2) - \mathbf{w}\cdot\mathbf{b}\right) = \partial_x\left(\lambda u\partial_x u + \kappa\partial_x\theta + \nu\mathbf{b}\cdot\partial_x\mathbf{b} + \mu\mathbf{w}\cdot\partial_x\mathbf{w}\right), \end{cases} \tag{1.1}$$

where $\rho(x,t)\in\mathbb{R}$, $u(x,t)\in\mathbb{R}$, $\mathbf{w}(x,t)\in\mathbb{R}^2$, $\mathbf{b}(x,t)\in\mathbb{R}^2$ and $\theta(x,t)\in\mathbb{R}$ denote, respectively, the mass density, longitudinal velocity, transverse velocity, transverse magnetic field and temperature of the fluids, and

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the longitudinal magnetic field is a constant which is taken to be one in (1.1). Here, constants $\lambda > 0$ and $\mu > 0$ are the viscosity coefficients of the fluids; constants $\nu > 0$ and $\kappa > 0$ are, respectively, the resistivity coefficient acting as the magnetic diffusion coefficient of the magnetic field and the heat conductivity coefficient. The total energy of the planar magnetohydrodynamics is

$$\mathcal{E} = \rho \left( e + \frac{1}{2}(u^2 + |\mathbf{w}|^2) \right) + \frac{1}{2}|\mathbf{b}|^2, \tag{1.2}$$

with the internal energy $e$ and the pressure $p$ which satisfy the following equations of state:

$$p = R\rho\theta, \qquad e = \frac{R}{\gamma - 1}\theta, \tag{1.3}$$

where $R > 0$ is a gas constant and $\gamma > 1$ is the adiabatic exponent. Roughly speaking, the system (1.1) arises from a three-dimensional MHD equations with a special structure: the flow depends on only one space variable $x \in \mathbb{R}$ and does not change in the transverse directions; however, the velocity and magnetic field still have three components. For the detailed derivation of planar MHD equations (1.1), please refer to [1–3] and references therein.

Recently, some important progress was made for the compressible non-isentropic MHD equations. A viscous profile for one-dimensional compressible MHD shock wave was given by a heteroclinic orbit of a six-dimensional gradient-like system of ordinary differential equations in [4]. For the initial–boundary value problem or free-boundary value problem, the existence, uniqueness, and regularity of global solutions for one-dimensional compressible MHD equations were established with large initial data by Chen and Wang in [1–3]. By using the method of weak convergence under certain (growth) conditions, the existence of global-in-time weak solutions to the three-dimensional compressible MHD equations was proved in [5]. See [6] for some interesting results on the vanishing shear viscosity limit for the one-dimensional compressible MHD equations. [7] proved the global existence of smooth solutions near the constant state for Cauchy problem to the three-dimensional compressible MHD equations by energy method and meanwhile, obtained convergence rates of the $L^p$-norm of these solutions to the constant state. Gao, Tao and Yao in [8] obtained the optimal decay rates for higher-order spatial derivatives of classical solutions near the constant state to the three-dimensional compressible MHD equations provided that the initial value satisfied the same assumption as [7]. Finally, we also mention that there are some results about the compressible isentropic MHD equations. Interested readers may refer to [9–13] and the references cited therein. So far, to our knowledge, there are no results about the large-time behavior of solutions to the compressible non-isentropic MHD equations (1.1) in a half line $\mathbb{R}_+ := (0, \infty)$. Here, we will partly give a positive answer for this important problem.

Initial data for system (1.1) is given by

$$(\rho, u, \mathbf{w}, \mathbf{b}, \theta)(x, 0) = (\rho_0, u_0, \mathbf{w}_0, \mathbf{b}_0, \theta_0)(x), \quad \inf_{x \in \mathbb{R}_+} \rho_0(x) > 0, \quad \inf_{x \in \mathbb{R}_+} \theta_0(x) > 0. \tag{1.4}$$

We assume that the initial data at the far field $x = +\infty$ is constant, namely

$$\lim_{x \to +\infty} (\rho_0, u_0, \mathbf{w}_0, \mathbf{b}_0, \theta_0)(x) = (\rho_+, u_+, \mathbf{w}_+, \mathbf{b}_+, \theta_+), \tag{1.5}$$

and the boundary data for $\rho$, $u$, $\mathbf{w}$, $\mathbf{b}$ and $\theta$ at $x = 0$ is given by

$$(\rho, u, \mathbf{w}, \mathbf{b}, \theta)(0, t) = (\rho_-, u_-, \mathbf{w}_-, \mathbf{b}_-, \theta_-), \quad \forall\, t \geq 0, \tag{1.6}$$

where $\rho_- > 0$, $u_- > 0$, $\theta_- > 0$ are constants, $\mathbf{w}_-$ and $\mathbf{b}_-$ are constant vectors, and the following compatibility conditions hold

$$\rho_0(0) = \rho_-, \quad u_0(0) = u_-, \quad \mathbf{w}_0(0) = \mathbf{w}_-, \quad \mathbf{b}_0(0) = \mathbf{b}_-, \quad \theta_0(0) = \theta_-. \tag{1.7}$$