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Variational–hemivariational approach to quasistatic viscoplastic contact problem with normal compliance, unilateral constraint, memory term, friction and damage☆

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A B S T R A C T

The aim of this paper is to study a mathematical model which describes the quasistatic frictional contact between a viscoplastic body and a foundation. The material's behavior is modeled with a rate-type constitutive law with internal state variable. The contact is modeled with normal compliance, unilateral constraint and memory term, friction and damage. We present the classical formulation of the problem and we derive its variational–hemivariational formulation. Finally, we prove its unique weak solvability. The proof is based on abstract results for a class of history-dependent variational–hemivariational inequalities.

1. Introduction

The goal of this paper is to study a frictional contact problem for rate-type viscoplastic materials. The materials' behavior is modeled with a constitutive law of the form

$$\dot{\sigma}(t) = \mathcal{E}\varepsilon(\dot{u}(t)) + \mathcal{G}(\sigma(t), \varepsilon(u), \zeta(t), \kappa(t)) \text{ in } \Omega \times (0, T), \tag{1}$$

where σ denotes the stress tensor, u is the displacement field, $\varepsilon(u)$ is the linearized strain tensor and $\zeta(t)$ is a damage function whereas κ denotes an internal state variable. Operator $\mathcal{E}$ represents the elastic properties of the material and $\mathcal{G}$ is a nonlinear constitutive function which describes its viscoplastic behavior. The mechanical models with internal state variable κ were considered in [1,2] where one can find various results, examples and mechanical interpretations in the study of viscoplastic materials. It is a vector-valued function

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which evolution is governed by the differential equation

$$\dot{\kappa}(t) = G(\sigma(t), \varepsilon(u(t)), \kappa(t)) \text{ in } \Omega \times (0,T), \tag{2}$$

where G is a nonlinear constitutive function which values are in $\mathbb{R}^m$ with m being a positive integer.

Dynamic contact problems for such materials were presented in [3–7]. On the other hand, quasistatic problems with various types of contact conditions have been studied in [8,9]. In papers of [8,9], the contact was frictionless and modeled with normal compliance, in [10,11] the contact was modeled with normal compliance and unilateral constraint, and in [12] the memory term was added to the contact condition. The contact is modeled with a multivalued friction law of the form

$$-\sigma_\tau(t) \in \partial j_\tau\big(\int_0^t \|u_\tau(s)\|_{\mathbb{R}^d} ds,\, u_\tau(t)\big) \text{ on } \Gamma_3 \times (0,T),$$

where ∂j_τ denotes a Clarke subdifferential of a function $j_\tau : \Gamma_3 \times \mathbb{R}^d \times \mathbb{R}^d to \mathbb{R}$ which is locally Lipschitz and, in general, nonconvex in its second variable. Here Γ_3 is the contact surface and u_τ represents the tangential part of the vector. Due to the nonmonotone character of the multivalued boundary conditions, there is no possibility to apply a convex analysis approach to the contact problem under consideration. This leads to a mathematical model that involves the Clarke subdifferential of a locally Lipschitz functional and as a consequence the model has form of hemivariational inequality.

One of the elements of weak formulation of the considered contact problem is a variational–hemivariational inequality which can be seen as the effective combination of the variational and hemivariational inequalities, in which both convex and nonconvex functionals are involved. These types of inequalities have been studied in [13–16]. The present paper represents a continuation of [16] where the quasistatic viscoplastic contact problem with normal compliance, unilateral constraint, memory term and friction was studied. The novelty in the present paper is the damage field which is involved in the model as well.

The paper is structured as follows. In Section 2 we present some preliminary material together with the notation. In Section 3 we describe the model of the contact process. Finally, in Section 4 we present the existence result for the weak formulation of the problem which consists of a system of equations and a hemivariational–variational inequality.

2. Preliminaries

In this section we recall basic definitions and notation needed in the sequel.

First, we recall that a function $h : X \to \mathbb{R}$ defined on a Banach space X is called locally Lipschitz, if for every $u \in X$ there exists a neighborhood $\mathcal{N}(u)$ of u such that

$$|h(y) - h(z)| \le K_u \|y - z\|_X \text{ for all } u \in \mathcal{N} \text{ with } K_u > 0.$$

The generalized directional derivative of Clarke of a locally Lipschitz function $h : X \to \mathbb{R}$ at $x \in X$ in the direction $v \in X$, denoted by $h^0(x; v)$, is defined by (cf. [17] Def 1)

$$h^0(x; v) = \limsup_{y \to x, \lambda \downarrow 0} \frac{h(y + \lambda v) - h(y)}{\lambda}.$$

The generalized gradient of a function $h : X \to \mathbb{R}$ at $x \in X$, denoted by $\partial h(x)$, is a subset of a dual space X^* given by

$$\partial h(x) = \{\zeta \in X^* : h^0(x; v) \ge \langle \zeta, v\rangle_{X^* \times X} \text{ for all } v \in X\}. \tag{3}$$

The convex subgradient of a convex function $h : X \to \mathbb{R}$ at $x \in X$, denoted by $\partial_c h(x)$, is a subset of a dual space X^* given by

$$\partial_c h(x) = \{\zeta \in X^* : h(x + v) - h(x) \ge \langle \zeta, v\rangle_{X^* \times X} \text{ for all } v \in X\}. \tag{4}$$