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Analysis of solutions for a cerebrospinal fluid model

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ABSTRACT

The aim of this manuscript is to analyze an intracranial fluid model from a mathematical point of view. The novel aspect of this work with respect to the existing literature is represented by the rigorous mathematical results regarding the analysis of the solutions to the systems of equations which model the different mechanisms in the cerebrospinal fluid compartments. We are able to prove the existence and uniqueness of a local solution and the existence and uniqueness of a global solution under some restriction conditions on the initial data. Moreover the last part of the paper is devoted to numerical simulations for the analyzed cerebrospinal fluid model. In particular, in order to assess the reliability of the stated theoretical results, we carry out the numerical simulations in two different cases: first, we fix initial data which satisfy the conditions for the global existence of solutions, then, we choose initial data that violate them.

1. Introduction

The main purpose of the present paper is to analyze an intracranial fluid model from a mathematical point of view. One of the main difficulties is related to the complexity of the intracranial dynamics which is the origin of many different phenomena: the flow of the cerebrospinal fluid throughout the CSF compartments, the mechanical interaction between the fluid and the brain, the physiology of the brain, the coupling with the circulatory-system and production and reabsorption laws.

There is a lack in literature of rigorous mathematical results regarding the analysis of the solutions to the systems of equations which model the different mechanisms in the cerebrospinal fluid compartments. Understanding the mathematical properties of the model is often intimately related to establishing its validity in the analysis of the physiological properties. Our aim here is then to provide a comprehensive study of the equations that rule the cerebrospinal fluid (CSF) dynamics.

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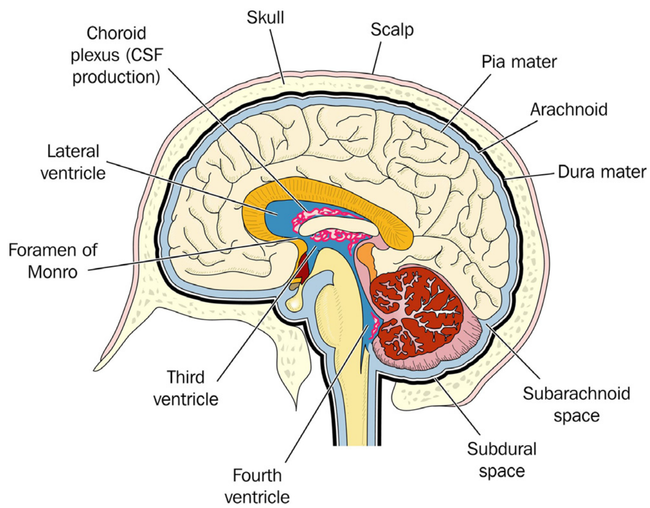

Fig. 1. Cross section of the human brain.

The CSF is derived by active secretion from cerebral arterial blood. The site of this process is only conceptually limited to the choroid plexus (see Fig. 1) of the ventricular system, a multi-lobed vascular structure of the pia mater that projects into the ventricles of the brain.

CSF, produced mainly in the lateral and third ventricles, flows along the aqueduct of Sylvius to reach the fourth ventricle. Passage through the narrow aqueduct is fast, and its pulsatile nature can be detected by precision dynamic MRI techniques. CSF flows out of the fourth ventricle through the midline foramen of Magendie and the lateral foramina of Luschka into subarachnoid space, which comprises a network of interconnected CSF cisterns located around the basal aspect of the brain. It flows upwards to the superior sagittal sinus where most of it is absorbed.

Drainage of CSF fluid into the venous compartment takes place predominantly (in humans) through arachnoid granulations that penetrate the walls of the sagittal sinus.

In order to make the CSF dynamics more comprehensible, detailed mathematical models quantifying forces and their interaction have become fundamental to reveal what medical instruments are not able to do without affecting the data.

Indeed, several models have been presented in the past decades, starting from the pioneering works by Marmarou and co-workers [1–3]. Some are especially aimed at analyzing individual aspects of intracranial dynamics, such as nonlinear cerebrospinal fluid production and reabsorption processes [4,5], nonlinear intracranial elasticity [6], and venous collapse [7]. More comprehensive multicompartmental approaches that simultaneously embody the relationships between intracranial elasticity, CSF circulation, and some features of cerebral hemodynamics have also been proposed [8–11].

More recently, Ursino et al. developed a model of craniospinal dynamics that incorporates most of the phenomena mentioned above [12,13].

Linninger et al. (see [14]) developed a lumped model for describing the CSF pulsatility during the cardiac cycle (see Fig. 2). This first model included the main constituents present in the cranial vault; however only the CSF compartment was fully treated. In fact the model includes the lateral ventricles, the third ventricle, the fourth ventricle and the subarachnoid space. The cerebral blood compartment is taken into account only as boundary condition for the CSF system. Indeed, the CSF pulsation is driven by a boundary condition modeling the effects of the choroid plexus which expands and contracts in a prescribed way (modeling the effect of the arterial pressure wave) and the CSF reabsorption is modeled by an equation where the venous