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Large time decay of solutions for the 3D magneto-micropolar equations

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ABSTRACT

This paper studies the global regularity and decay estimates of solutions to the three-dimensional (3D) magneto-micropolar equations. We establish three main results. The first result is the optimal decay rates in L^2 of the weak solutions of 3D magneto-micropolar equations with large initial data, where to prove this result we need to overcome the difficulty that comes from the presence of linear terms. The second result is the existence and uniqueness of global smooth solutions of the equations with small initial data. Then based on these two results, and using Fourier splitting method, we obtain our third main result, namely, the decay rates in L^2 for higher order derivatives of the smooth solution with small initial data.
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1. Introduction

The 3D incompressible magneto-micropolar fluid equations can be written as

$$\begin{cases} \partial_t u + u \cdot \nabla u = -\nabla p + (\mu + \chi)\Delta u + b \cdot \nabla b + 2\chi \nabla \times \omega, \\ \partial_t \omega + u \cdot \nabla \omega - \kappa \nabla \nabla \cdot \omega + 4\chi\omega = \gamma \Delta \omega + 2\chi \nabla \times u, \\ \partial_t b + u \cdot \nabla b = \nu \Delta b + b \cdot \nabla u, \\ \nabla \cdot u = 0, \ \nabla \cdot b = 0, \\ u(x,0) = u_0(x), \omega(x,0) = \omega_0(x), b(x,0) = b_0(x), \end{cases} \tag{1.1}$$

where $x \in \mathbb{R}^3$ and $t \geq 0$, u, ω, b and p denote the velocity of the fluid, micro-rotational velocity, the magnetic field and the hydrostatic pressure respectively. μ, χ and $\frac{1}{\nu}$ are, respectively, kinematic viscosity, vortex viscosity and magnetic Reynolds number. κ and γ are angular viscosities.

When the magnetic field is absent ($b = 0$) and there is no micro-structure ($\chi = 0$), then system (1.1) reduces to 3D Navier–Stokes equations. The global regularity problem of 3D Navier–Stokes equations is a millennium prize problem and still an extremely challenging problem. In [1], Leray posed an important

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open problem, namely, whether or not the global weak solutions of this system decay to zero in L^2 as time tends to infinity. This is a nontrivial problem. Until 1984, Kato [2] has established the decay for the strong solutions with initial data small. By introducing the method of Fourier splitting, the algebraic decay rate for weak solutions was obtained first by Schonbek [3], and the problem posed by Leray was resolved. Later, the result in [3] is sharpened and extended in [4], see also [5–8]. Recently, Schonbek and Wiegner [9] established the decay estimates of the higher order derivatives of smooth global solution with small initial data.

If there is no micro-structure ($\chi = 0$), the velocity u does not depend on the micro-rotation field ω, then system (1.1) becomes magnetohydrodynamic (MHD) equations, which can be used to model the magnetic properties of electrically conducting fluids such as plasmas, liquid metals, salt water (see, e.g., [10]). The well-posedness problem of MHD equations has attracted considerable attention from the community of mathematical fluids (see, e.g., [11–17]). Moreover, many authors have studied MHD equations from the point of view of long time behavior. Without making a complete list of all authors we would like to mention some of the relevant literature. Several decay interesting results can be found in [18–22], where the decay estimate in L^2 for weak solutions themselves and the higher order derivatives of global strong solutions with initial data small were obtained.

As only the magnetic field disappears ($b = 0$), then system (1.1) reduces to 3D incompressible micropolar equations. This system was first introduced by Eringen [23], which models the micropolar fluids such as fluids consisting of particles suspended in a viscous medium. The global existence of weak solutions and strong solutions with initial data small for 3D micropolar equations were obtained in [24,25]. The time decay rates was obtained by Chen and Price [26] for small L^3 strong solutions of the 3D micropolar equations (1.1) via Kato's method. To the best of the author's knowledge, no further investigation on the L^2 time decay properties for the 3D incompressible micropolar equations (1.1) has been pursued. For the 2D case, based on the spectral decomposition of linearized micropolar fluid flows, there have been several important developments in this direction, see [27–29] for example.

The full system (1.1) can be used to model the motion of a micropolar fluid which is moving in the presence of a magnetic field (see, e.g., [30]). For the 2D magneto-micropolar equations (1.1), quite a few important global regularity results are available (see, e.g., [31–37]). For the 3D case, the global regularity problem is still an important open problem. The local existence of strong solution, global existence of strong solutions for small initial data and the global existence and uniqueness of weak solutions were obtained in [38–40] respectively. Recently, the global existence of smooth solutions and the decay estimates of solutions to the 3D compressible magneto-micropolar equations were established under the condition that the initial data are small perturbations of some given constant state in [41].

In this paper we study the existence and the decay estimates of solutions of the 3D magneto-micropolar equations (1.1). Motivated by the ideas in [3,4,9], we will show the decay estimates in L^2 for the weak solutions and the higher order derivatives of the smooth solutions with small initial data of system (1.1).

Before stating our main results, we first give the definition of the weak solution of (1.1). In the following, C_w denotes the continuity in the weak L^2 sense.

Definition 1.1. Let $T > 0$. A function

$$(u, \omega, b) \in C_w([0, T]; L^2(\mathbb{R}^3)) \cap L^2(0, T; H^1(\mathbb{R}^3))$$

is called a weak solution of (1.1), if (u, ω, b) satisfies the following conditions:

(i) For any $\varphi \in C_0^\infty(\mathbb{R}^3 \times [0, T])$ satisfying $\nabla \cdot \varphi = 0$ and for a.e. $t \in [0, T]$,

$$\begin{aligned}
&\int_{\mathbb{R}^3} u(x,t) \cdot \varphi(x,t) dx - \int_{\mathbb{R}^3} u_0(x) \cdot \varphi(x,0) dx - \int_0^t \int_{\mathbb{R}^3} u \cdot \partial_\tau \varphi dx d\tau \\
&= (\mu + \chi) \int_0^t \int_{\mathbb{R}^3} u \cdot \Delta \varphi dx d\tau + \int_0^t \int_{\mathbb{R}^3} (u \otimes u) : \nabla \varphi dx d\tau
\end{aligned}$$

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