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Positive steady states for a prey–predator model with population flux by attractive transition

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A B S T R A C T

This paper studies the stationary solutions of a prey–predator model with population flux by attractive transition. We first obtain a bifurcation branch (connected set) of positive solutions which connects two semitrivial solutions. Next we derive the asymptotic behavior of positive solutions as the coefficient α of the population flux tends to infinity. A main result implies that positive solutions can be classified into two types as $\alpha \to \infty$. In one type of them, as $\alpha \to \infty$, positive solutions of the prey–predator model approach positive solutions of a competition model with equal diffusion coefficients.

1. Introduction

This paper is concerned with the following diffusive Lotka–Volterra prey–predator model in a bounded habitat $\Omega(\subset \mathbb{R}^N)$ with smooth boundary $\partial\Omega$:

$$\begin{cases} u_t = d_1 \Delta u + u(m_1 - u - cv), & (x,t) \in \Omega \times (0,T), \\ v_t = \nabla \cdot \left[d_2 \nabla v + \alpha u^2 \nabla \left(\dfrac{v}{u} \right) \right] + v(m_2 + bu - v), & (x,t) \in \Omega \times (0,T), \\ u = v = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) \geq 0, \quad v(x,0) = v_0(x) \geq 0, & x \in \Omega. \end{cases}$$

Here unknown functions $u(x,t)$ and $v(x,t)$ represent population densities of the prey and the predator at location $x \in \Omega$ and time $t > 0$, respectively; d_i $(i = 1,2)$ are coefficients of the linear diffusion and m_i

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($i = 1, 2$) are growth rates of each species, where d_1, d_2 and m_1 are positive constants but m_2 is a real constant which is allowed to be negative. The positive coefficients b and c denote the rate of increase of the predator and the rate of decrease of the prey due to the predation, respectively. In this model, the diffusion of the predator is determined by the population flux

$$-d_2\nabla v - \alpha u^2\,\nabla\left(\frac{v}{u}\right).$$

The former term $-d_2\nabla v$ corresponds to a usual Fickian diffusion depending only on the distribution of the predator, whereas the latter term

$$\boldsymbol{J} := -\alpha u^2\,\nabla\left(\frac{v}{u}\right) = \alpha(-u\nabla v + v\nabla u)$$

describes the population flux of the predator based on a biodiffusion such that the transition probability of each individual of the predator depends on conditions at the point of arrival (Okubo and Levin [1, Section 5.4]). That is to say, each individual of the predator has a stochastic tendency to move towards the high density area of the prey. The nonnegative constant α is a magnitude of such a population flux by attractive transition. Here it should be noted that, from the viewpoint of the ecological modeling, the above flux $\boldsymbol{J}$ differs from the so-called chemotaxis term $\alpha v\nabla u$ in which the transition probability of each individual of the predator depends on the difference of feed (prey) between at arrival and at departure. As far as we know, there are few works focusing on the mathematical effect of the population flux $\boldsymbol{J}$ in the research field of the diffusive Lotka–Volterra equations.

As the first step of the mathematical analysis for the effect of $\boldsymbol{J}$, this paper focuses on the effect on the stationary solutions. The stationary problem consists of the nonlinear elliptic equations

$$d_1\Delta u + u(m_1 - u - cv) = 0 \quad \text{in } \Omega, \tag{1.1a}$$

$$\nabla\cdot\left[d_2\nabla v + \alpha u^2\,\nabla\left(\frac{v}{u}\right)\right] + v(m_2 + bu - v) = 0 \quad \text{in } \Omega, \tag{1.1b}$$

subject to the homogeneous Dirichlet boundary conditions

$$u = v = 0 \quad \text{on } \partial\Omega \tag{1.1c}$$

and the nonnegative conditions

$$u \geq 0 \quad \text{and} \quad v \geq 0 \quad \text{in } \Omega. \tag{1.1d}$$

Our analysis for (1.1) will use not only (1.1b) but also the following equivalent form:

$$d_2\Delta v + \alpha(u\Delta v - v\Delta u) + v(m_2 + bu - v) = 0 \quad \text{in } \Omega \tag{1.2}$$

or the following semilinear form

$$\Delta v + \frac{v}{d_2 + \alpha u}\left(\frac{\alpha u}{d_1}(m_1 - u - cv) + m_2 + bu - v\right) = 0 \quad \text{in } \Omega, \tag{1.3}$$

which is obtained by substituting (1.1a) into (1.2).

It should be noted that the classical diffusive prey–predator model with $\alpha = 0$ has been extensively studied by a lot of mathematicians (e.g., [2–13]).

In what follows, we call (u, v) a positive solution if (u, v) is a solution of (1.1) satisfying $u > 0$ and $v > 0$ in Ω. From the ecological viewpoint, positive solutions of (1.1) are corresponding to coexistence steady states of the prey and the predator. The purpose of this paper is to derive

- the global bifurcation structure of positive solutions of (1.1)

and

- the asymptotic behavior of positive solutions as α tends to infinity.