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Optimal harvesting for a size-stage-structured population model[☆]

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ABSTRACT

In this paper, we consider the optimal harvesting for a size-stage-structured population model. The main feature of this article is to study optimal harvesting, stage structure and body size at the same time in a single population system. At first, under appropriate assumptions, by the method of characteristic and simple changes of variables, we obtain the properties of solutions of our model, including the global existence and uniqueness, nonnegativity and boundedness. Then, we give several useful estimates about the solutions. Taking advantage of these estimates and relevant theory of functional analysis, we derive the existence of optimal harvesting strategy. Finally, a discussion of related problems is also presented.

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1. Introduction

The purpose of this work is to investigate the existence of optimal harvesting for a size-stage-structured population model

$$\begin{cases} \partial_t j(x,t) + g(A(t))\partial_x j(x,t) = -\beta j(x,t), \quad x \in [0,m],\ t > 0, \\ A'(t) = -v(t)A(t) - h(t)A(t) + g(A(t))j(m,t), \quad t > 0, \\ g(A(t))j(0,t) = \gamma A(t), \quad t > 0, \\ j(x,0) = j_0(x), \quad x \in [0,m], \\ A(0) = A_0. \end{cases} \tag{1.1}$$

As far as (1.1) is concerned, it is necessary to make the following instructions.

◊ The state variable $j(x,t)$ denotes the population density of juvenile individuals with size $x \in [0,m]$ at time t; $m \in (0,l)$ is the required level of size for adulthood and the juvenile population must accumulate a sufficient amount to enter adulthood, $l \in (0,\infty)$ is the maximal size. The positive constant β is the death rate

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of juvenile population. Accordingly, the total population at time t of members of juvenile individuals between size 0 and m is $\int_0^m j(x,t)dx$. $g(A(t))$ represents the growth rate in size of the juvenile population depending on the quantity of adult population $A(t)$. The first equation of model (1.1) is called the McKendrick equation (1926), see [1,2]. The initial size distribution of juvenile individuals is $j_0(x)$, which is a known function of size x. The third equation of (1.1) is a boundary condition, the expression $g(A(t))j(0,t)$ stands for the birth rate of juvenile population of size 0 at time t relating to the quantity of adult population, the positive constant γ represents the reproductive rate of adult population.

$\Diamond$ The function $A(t)$ is the number of adult population at time t whose body size belongs to $[m,l)$, the corresponding initial value of adult population is $A_0 \geq 0$. The function $v(t)$ denotes the death rate of adult population depending on time, the function $h(t)$ represents the harvesting rate of adult population. The term $g(A(t))j(m,t)$ in the second equation of (1.1) means that the juvenile individuals accumulated the sufficient size $x = m$ enter adulthood.

Throughout this article, our assumptions are as follows.

$(\mathcal{G})$ $g : [0,\infty) \to (0,\infty)$ is continuously differentiable satisfying

$$\lim_{u\to\infty} g(u) = 0, \quad g'(u) < 0, \quad \forall u \in [0,\infty).$$

$(\mathcal{V})$ $v : [0,\infty) \to (0,\infty)$ is Lipschitz continuous, and there exist v_*, $v^* > 0$ such that

$$v_* \leq v(t) \leq v^*, \quad \forall t \in [0,\infty).$$

$(\mathcal{H})$ $h : [0,\infty) \to (0,\infty)$ is Lipschitz continuous, and there exist $\underline{h}$, $\overline{h} > 0$ such that

$$\underline{h} \leq h(t) \leq \overline{h}, \quad \forall t \in [0,\infty).$$

$(\mathcal{J})$ $j_0 : [0,m] \to (0,\infty)$ is a Lipschitz continuous function.

Control variable in our paper is harvesting rate $h(t)$. From the properties of $h(t)$, it is easy to show that $h(t)$ belongs to admissible set of controls

$$\Omega = \{h \in L^\infty(0,T) \mid 0 < \underline{h} \leqslant h(t) \leqslant \overline{h},\ a.e.\ t \in [0,T]\},$$

where T is the finite sufficient horizon of control. In addition, $L^\infty(0,T)$ denotes the Banach space of essentially bounded functions with uniform norm. $(j^h(x,t), A^h(t))$ is the solution of system (1.1) corresponding to $h \in \Omega$. Combining the economic benefits and the sustainable development of renewable resources, the optimal harvesting problem we study here is formulated as

$$\text{Max} \quad E(h) := \int_0^T p \cdot h(t)A^h(t)dt - \int_0^T \int_0^m q \cdot j^h(x,t)dxdt, \quad h \in \Omega, \tag{1.2}$$

where positive constant p is the economic value of an adult individual, positive constant q represents the cost for implementing the cultivation and protection of juvenile individuals after the harvesting of adult individuals. Hence, $E(h)$ stands for the total economic benefits obtained from the harvesting process in $[0,T]$. The aim of this problem is to find the control $h^* \in \Omega$ which realizes the maximum of $E(h)$, then this control h^* is the optimal harvesting strategy.

Along with the socio-economic development, the ecological resources are faced with being exhausted because of human over-exploitation. However, with the progress of society and attention to sustainable development, people gradually realize that the ecological resources not only provide the direct economic value, but also have an extremely vital ecological value. During the last few decades, the research of population resources development and management has been quickly developed and played an important

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