

Turing instability and formation of temporal patterns in a diffusive bimolecular model with saturation law

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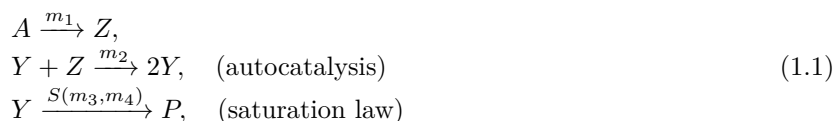
ABSTRACT

In the present article, a bimolecular chemical reaction–diffusion model with autocatalysis and saturation law is considered. The local asymptotic stability and instability of the unique feasible equilibrium of the local system, and the existence of Hopf bifurcation of the local system at this unique equilibrium are analyzed in detail. In the stability domain of the equilibrium of the local system, the effect of the spatial diffusion including the variation of the size of the space domain and the diffusion coefficient on the stability is studied and Turing instability is demonstrated. In the instability domain of the local system, time-periodic patterns of the original reaction–diffusion system bifurcating from the constant positive steady state are found according to the Hopf bifurcation theorem for reaction–diffusion dynamical systems with homogeneous Neumann boundary conditions by considering various different bifurcation parameters. Finally, to verify the obtained theoretical prediction, some numerical simulations are also included.

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1. Introduction

It is well known that the bimolecular chemical reaction processes should be described by the reaction–diffusion system with autocatalysis and saturation law. Consider the reaction process given by (see [1–5])



where A, Y, Z and P are the chemical reactants and products; m_1, m_2, m_3 and m_4 represent the reaction rates and the function $S(m_3, m_4)$ accounts for the saturation laws often used like the Langmuri–Hinshelwood law in heterogeneous catalysis and adsorption, the Michaelis–Menten law in enzyme-controlled processes and the Holling law in ecology.

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Assume that, in the reaction process (1.1), the in-and-out-flow of A and P is open, all the reaction steps are irreversible and the concentrations of A and P are independent of time and spatial variables, that is, they are kept uniform throughout the reaction. As well, assume that the saturation law in the reaction process (1.1) follows the one of Michaelis–Menten types and there is no flux across the boundary of the reactor. If we consider only the isothermal processes but not consider the convective phenomenon, then the reaction process (1.1) can be described by the following reaction–diffusion model with autocatalysis and saturation law in the form

$$\begin{cases} \frac{\partial U}{\partial T} = D_1 \Delta U + m_2 UV - \frac{m_3 U}{1 + m_4 U}, & X \in \Omega, T > 0, \\ \frac{\partial V}{\partial T} = D_2 \Delta V + m_1 A_0 - m_2 UV, & X \in \Omega, T > 0, \\ \frac{\partial U}{\partial \nu} = \frac{\partial V}{\partial \nu} = 0, & X \in \partial\Omega, t > 0, \\ U(X, 0) = U_0(X) \geq 0, V(X, 0) = V_0(X) \geq 0, & X \in \bar{\Omega}, \end{cases} \quad (1.2)$$

where Δ is the Laplace operator and the bounded domain Ω in $\mathbb{R}^n (n \geq 1)$ with a smooth boundary $\partial\Omega$ denotes the reactor; ν is the outer unit normal vector on $\partial\Omega$ and the homogeneous Neumann boundary condition is imposed because there is no flux across the boundary $\partial\Omega$; A_0, U and V represent respectively the concentrations of reactants A, Y, Z ; the positive constants D_1 and D_2 stand for the diffusion coefficients of reactants Y and Z , respectively; $m_1, m_2, m_3 > 0$ are the reaction rates and $m_4 > 0$ accounts for the strength of the saturation law.

Let $t = m_3 T, x = \sqrt{\frac{m_3}{D_1}} X$ and introduce the new variables $u(x, t)$ and $v(x, t)$ by

$$\begin{aligned} u(x, t) &= \frac{m_2}{m_3} U(X, T) = \frac{m_2}{m_3} U \left(\sqrt{\frac{D_1}{m_3}} x, \frac{t}{m_3} \right), \\ v(x, t) &= \frac{m_3}{m_1 A_0} V(X, T) = \frac{m_3}{m_1 A_0} V \left(\sqrt{\frac{D_1}{m_3}} x, \frac{t}{m_3} \right). \end{aligned}$$

Then the system (1.2) is reduced to the following one

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + auv - \frac{u}{1 + mu}, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = \sigma \Delta v + 1 - uv, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & X \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) \geq 0, v(x, 0) = v_0(x) \geq 0, & x \in \bar{\Omega}, \end{cases} \quad (1.3)$$

where the scaled spatial domain is still denoted by Ω and

$$a = \frac{m_1 m_2 A_0}{m_3^2}, \quad m = \frac{m_3 m_4}{m_2} \quad \text{and} \quad \sigma = \frac{D_2}{D_1}.$$

System (1.3) has been investigated extensively by many researchers and some interesting results also have been obtained, see [3–5]. For instance, Peng, Shi and Wang [4] considered the existence and non-existence of non-constant steady-state solutions of (1.3) when the diffusion rate σ is large or small by using the degree theory. In [3], Yi et al. studied the spatiotemporal patterns of (1.3) with $\sigma = 1$ and the spatial domain Ω taken as the one-dimensional interval $(0, \ell\pi)$ with $\ell > 0$. Lately, Peng and Yi in [5] improved the result of steady state bifurcation obtained in [3] and investigated further the effect of various parameters on spatiotemporal patterns. As one of the mechanisms of pattern formation, the spatially nonhomogeneous steady states

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