



Effects of a degeneracy in a diffusive predator–prey model with Holling II functional response[☆]

Shanbing Li^{a,*}, Jianhua Wu^b, Yaying Dong^c

^a School of Mathematics and Statistics, Xidian University, Xi'an, 710071, PR China

^b College of Mathematics and Information Science, Shaanxi Normal University, Xi'an, 710062, PR China

^c School of Science, Xi'an Polytechnic University, Xi'an 710048, PR China

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ABSTRACT

In this work, we study a spatially heterogeneous predator–prey model where the interaction is governed by a Holling II functional response. The existence of coexistence states of the two species by using the bifurcation theory is discussed. As a result, it is shown that the degeneracy for the prey and the predator has distinctly different effects on the coexistence states of the two species when the intrinsic growth rate of the prey is above a certain critical value. We also study the asymptotic behavior of positive stationary solutions as the intrinsic growth rate of the predator tends to a certain critical value.

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1. Introduction

A variety of reaction–diffusion equations are used to describe some phenomena arising in population ecology. In numerous reaction–diffusion equations, the predator–prey model is an important branch. Combining the growth, interaction and dispersal of the predator and prey, a natural mathematical model for their spatiotemporal behavior is the following predator–prey system for two species of the form

$$\begin{cases} u_t - d_1(x)\Delta u = \lambda(x)u - a(x)u^2 - b(x)\phi(u)v, & x \in \Omega, t > 0, \\ v_t - d_2(x)\Delta v = \mu(x)v - d(x)v^2 + c(x)\phi(u)v, & x \in \Omega, t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) \geq 0, v(x, 0) = v_0(x) \geq 0, & x \in \partial\Omega. \end{cases} \quad (1.1)$$

Here we assume that both the predator and prey populations have a logistic growth rate. The function $\phi(u)$ represents the functional response of the predator. As known to all, the classical Lotka–Volterra model

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* Corresponding author.

E-mail address: lishanbing@xidian.edu.cn (S. Li).

assumes that $\phi(u) = u$. But when the handling time of each prey is also considered, a more reasonable response function is the Holling type II response $\phi(u) = u/(1 + mu)$ for some $m > 0$, which was first examined by Holling [1].

The dependence on the space variable x of these coefficient functions represents the fact that the prey u and predator v interact in a spatially heterogeneous environment. If the environment is spatially homogeneous, then all these coefficient functions reduce to positive constants, and (1.1) is known as, in this special case, the classical predator–prey model with diffusion, which has attracted extensive study. For example, J. López–Gómez and R. Pardo [2] studied the existence of coexistence states for the classical diffusive predator–prey model. Furthermore, they established the uniqueness of a coexistence state for the one-dimensional prototype model in [3,4]. In [5], the existence and uniqueness of coexistence states for a predator–prey model with Dirichlet boundary conditions are studied. The reference [6] was the first paper dealing with a quasilinear predator–prey model. Y. Yamada [7] studied the stability of steady-states for prey–predator diffusion equations with homogeneous Dirichlet conditions. Du and Lou [8,9] established some uniqueness and exact multiplicity results for a predator–prey model. For more results, one can see [10–17] and the references therein. It is interesting to know whether the model behaves differently when the environment is spatially heterogeneous. A limiting case is when some of the coefficient functions in (1.1) vanish partially over Ω , which we henceforth call a degeneracy. [18,19] are the two most pioneering papers in the theory of competing species models introducing degeneracies, or refuge areas, in their coefficients. For the single degenerate equation, in the past few decades, J. López–Gómez and collaborators built new methods and obtained many interesting results in [20–26], these results are different from the classical single equation. For the prey–predator or competition model with a degeneracy, the influence of certain spatial heterogeneity can cause significant changes of dynamic behavior. One can see [27–33] and the references therein for more information. Finally, we point out that there are some strong asymmetries according to the coefficients where the degeneration is placed had been already observed in the context of competing species models (compare [28,29] with [25] and Chapter 10 of [26]). The more information about the differences between them is stated in the last paragraph of Chapter 10 of [26].

As in [30,34], if the degeneracy occurs in the prey equation of (1.1), then the dynamical behavior of (1.1) changes drastically from the classical model (see Theorems 2.1 and 2.2 for details). The main purpose of this paper is to show that the dynamical behavior of (1.1) also changes drastically from the classical model if the degeneracy occurs in the predator equation of (1.1), moreover, the change is fundamentally different from that in [30,34] (see Theorems 2.3 and 2.4 for details).

The rest of this paper is organized as follows. In Section 2, we state the main results of this paper. In Section 3, we show some non-existence result and a priori estimates of positive solutions. In Section 4, we obtain positive solutions from the viewpoint of the local bifurcation theory. In Section 5, we accomplish the proof of our main results.

2. Main results

For (1.1), J. López–Gómez and collaborators have studied the existence and uniqueness of coexistence states with constant coefficients under Dirichlet boundary conditions [5], moreover, a very general version of this model, with spatially heterogeneous coefficients, under general boundary conditions and general second order elliptic operators, was introduced and analyzed on Section 7.2 of [35]. As a particular case, the authors [30,34] further considered the following predator–prey model with degeneracy in the prey equation:

$$\begin{cases} -\Delta u = \lambda u - a(x)u^2 - \frac{buv}{1 + mu}, & x \in \Omega, \\ -\Delta v = \mu v - v^2 + \frac{cuv}{1 + mu}, & x \in \Omega, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, \end{cases} \quad (2.1)$$

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