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Threshold dynamics and optimal control of an age-structured giving up smoking model

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HIGHLIGHTS

- A new giving up smoking model with the continuous age-structure in the chain smokers class is proposed.
- Local and global stabilities are studied to establish strict threshold dynamics.
- The optimal control strategy on potential smokers is proposed.
- Numerical simulations on dynamical results and optimal control are performed.

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ABSTRACT

In this paper, a new giving up smoking model is proposed by incorporating the continuous age-structure in the chain smokers class, which is known as a class of age-structured giving up smoking model. Smoking generation number is defined and proved to be a classic threshold parameter. Two steady states of the model are found. The corresponding characteristic equations are analyzed, and the proper Lyapunov functionals are constructed to show the local and global stability of both steady states. Optimal control strategy is presented and numerical simulations are made to support the main results.

1. Introduction

The World Health Organization reported that smoking causes 250 million children and adolescent deaths, and predicted that 10 million people will die of smoking-related diseases every year by 2030 [1]. Smoking-related diseases include heart disease, lung cancer, emphysema and chronic bronchitis [2]. Therefore, smoking

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use is considered as a significant global public health problem. Similar with the spread of many infectious diseases, smoking can spread through social contact. Thus, mathematical modeling has been widely used to investigate the dynamics of smoking. The first giving up smoking mathematical model was proposed by Castullo-Garsow et al. in [3], where they divided the population into three classes: potential smokers (P), smokers (S) and quit smokers (Q). The authors concluded that smoking-free equilibrium is globally asymptotically stable when the smoking generation number is less than 1 and unstable when it is greater than 1. The local stability was proved by using the Routh–Hurwitz criterion while the global stability was only conjectured by using numerous simulations. Subsequently, Sharomi and Gumel [4] introduced a new category Q_t of smokers who temporarily give up use of smoking. Afterward many mathematicians explored different aspects of giving up smoking model. To understand the dynamical behavior and other features of giving up smoking model in more detail, the readers can refer to the monographs [5–8] and the references cited. It should be noticed that almost all giving up smoking models are described by ordinary differential equations (ODE) or fractional order differential equations (FODE). For fractional order giving up smoking model, the readers are advised to see a recent work of Singh et al. [9] where the authors proved the existence and uniqueness of the solution. However, they have not taken into account an important factor of transmission coefficient and the recover rate from chain smoking depending on the age of chain smokers. This suggests that ODE models may not be reasonable for understanding the dynamics of giving up smoking model. It is necessary to consider the age-structured giving up smoking. We refer the readers to the famous books on age-structured models, see Webb [10] and Iannelli [11], etc.

Although age-structured epidemic models have been studied widely [12–16], there are few papers concerning the influence of age factor on the dynamics of giving up smoking model. For instance, whether the threshold property of smoking generation number can be preserved or not, whether the optimal control strategies exist or not. Smoking frequency and amount are closely associated with age at smoking initiation. Also it has been found that earlier smoking initiation among adolescents expedite the likelihood of becoming frequent, daily, and chain smokers. Therefore, preventing or delaying smoking is important using proper smoking prevention programs. Based on the results of this study, it is suggested that smoking prevention programs should be initiated from young adolescent period. To study the relevancy of age and smoking in more detail, we refer to see the surveys [17–19] and the references cited.

It is worthy to notice that most of the authors cited above have investigated a number of giving up smoking ODE/FODE models and presented some analysis therein. However, they ignored age effect and thus differences between these models and realistic data are divine. It will be more realistic if we include age-structured in modeling the dynamics of giving up smoking model. Motivated by the above consideration, we proposed a new giving up smoking model with continuous age-structure in the chain smokers class. First, we proceed to propose a new giving up smoking model with continuous age-structure in the chain smokers class. Then, in order to analyze the dynamics, we make some preliminaries, including the existence of steady states and asymptotic smoothness of the semi flow. Furthermore, we employ the Volterra integral equations, analyze the corresponding characteristic equations, and construct the Lyapunov functionals to prove the uniform persistence, local stability and global stability, respectively. Finally, we present an optimal control strategy and numerical simulations to support our main results.

The proposed model is more generalized than the models already presented in the literature, for example, the model covers the classical giving up smoking models [4,5,7] as special cases. Since we have included age factor into the proposed model, therefore, the results presented here are more realistic as shown in the simulation section. Moreover, the current study will provide a platform for the researchers to include age-structure in other classes too and how to control smoking in a community. Also, it will help the policymakers to select an effective control strategy and apply them in a most cost-effective way. The remaining parts of this paper are organized as follows. In Section 2, a new giving up smoking model is proposed. Preliminaries are discussed in Section 3. The threshold dynamics of the proposed model, such as uniform persistence, local