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Hyperbolic hemivariational inequalities controlled by evolution equations with application to adhesive contact model[☆]

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ABSTRACT

A system which couples an abstract hemivariational inequality of hyperbolic type and an evolution equation in a Banach space is studied. The global existence of the system is established by exploiting the Rothe method. An application to a dynamic adhesive viscoelastic contact problem with friction is provided for which results on existence and regularity of weak solutions are proved.

1. Introduction and preliminaries

In this paper we study the solvability of a coupled system which consists of an abstract evolution hemivariational inequality and an ordinary differential equation in a Banach space. Our goal is to provide a global (in time) existence result without any smallness assumption. The system serves as a model in numerous applications in mechanics, physics and engineering. We provide an illustration of our abstract results and study a dynamic frictional contact problem with adhesion for viscoelastic materials. In this problem the hemivariational inequality describes the displacement field and the ordinary differential equation is for the adhesion field.

Adhesion processes are important in several industrial settings where various components are glued together. In the study of such processes, composite materials may undergo delamination under stress, in

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which different layers debond and move relative to each other. For this reason to obtain more precise models of contact phenomena, there is a need to add adhesion process to the description of contact. In this paper, we adopt the approach of Frémond [1,2] and introduce a surface internal variable, the bonding field, which takes values between zero and one, and which describes the fraction of active bonds on the contact surface. Note that various mathematical results for dynamic processes of adhesive contact between a deformable body and a foundation can be found in many contributions, see, for example, [3–14].

Furthermore, we mention that the theory of hemivariational inequalities is based on properties of the Clarke subgradient, defined for locally Lipschitz functions. It has started with the works of Panagiotopoulos, see [15,16] and has been developed during the last thirty years. The mathematical results on hemivariational inequalities have found many applications, see [17–21] and the references therein.

The main novelties of the paper are described as follows. First, for the first time, we apply the Rothe method to study a system of a hemivariational inequality and a differential equation. Until now, there are a few papers devoted to the Rothe method for hemivariational inequalities, see [22–26]. All of them studied only a single hemivariational inequality.

Second, our results are applied to a special case of Problem 6 (see Section 2) in which the locally Lipschitz functional J is independent of the adhesion field β and this problem reduces to the following hyperbolic hemivariational inequality: find $u \in \mathcal{V}$ such that $u' \in \mathcal{W}$, $u(0) = u_0$, $u'(0) = v_0$ and

$$\langle u''(t) + Au'(t) + Bu(t) - f(t), v\rangle + J^0(Mu(t); Mv) \geq 0 \tag{1}$$

for all $v \in V$, a.e. $t \in (0, T)$. Existence of solution to (1) has been proved earlier in [27] by using a surjectivity result for pseudomonotone operators under a suitable smallness condition. Now, we remove the smallness condition and use another approach than that in Theorem 6 in [27].

Third, note that Problem 6 has been recently studied in [3] under the assumption $c_2 - 2mc_e^2 T\|\gamma\| > 0$ (see (11) in [3]). This means that the existence result provided there is only local in time. In this paper, we overcome this flaw and obtain a global existence result for Problem 6. Furthermore, we also provide a method to construct a sequence which converges to a solution of Problem 6. The uniqueness of solutions to Problem 6 is an interesting open problem.

In the rest of this section we shortly recall some results which are needed in the sequel, see [20,28–30].

Let Y be a reflexive Banach space and $\langle \cdot, \cdot\rangle$ denote the duality of Y^* and Y. An operator $A : Y \to Y^*$ is pseudomonotone if for each sequence $\{y_n\} \subseteq Y$ converging weakly to $y \in Y$ such that $\limsup\langle Ay_n, y_n - y\rangle \leq 0$, we have $\langle Ay, y - z\rangle \leq \liminf\langle Ay_n, y_n - z\rangle$ for all $z \in Y$. It is known, see [20, Proposition 3.66], that the operator $A : Y \to Y^*$ is pseudomonotone if and only if the conditions $y_n \to y$ weakly in Y and $\limsup\langle Ay_n, y_n - y\rangle \leq 0$ imply $\lim\langle Ay_n, y_n - y\rangle = 0$ and $Ay_n \to Ay$ weakly in Y^*. It is also easy to check that if an operator is linear, bounded and nonnegative, then it is pseudomonotone.

Definition 1. Let Y be a reflexive Banach space. An operator $T : Y \to 2^{Y^*}$ is pseudomonotone if

(a) for every $v \in Y$, the set $Tv \subset Y^*$ is nonempty, closed and convex.

(b) T is upper semicontinuous from each finite dimensional subspace of Y to Y^* endowed with the weak topology.

(c) for any sequences $\{u_n\} \subset Y$ and $\{u_n^*\} \subset Y^*$ such that $u_n \to u$ weakly in Y, $u_n^* \in Tu_n$ for all $n \geq 1$ and $\limsup\,\langle u_n^*, u_n - u\rangle \leq 0$, we have that for every $v \in Y$, there exists $u^*(v) \in Tu$ such that $\langle u^*(v), u - v\rangle \leq \liminf_{n\to\infty}\langle u_n^*, u_n - v\rangle$.

The following result, see [29, Proposition 1.3.66], provides a criterion for the pseudomonotonicity of an operator, which will be used in Section 2.