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Homogenization of nonisothermal immiscible incompressible two-phase flow in porous media

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ABSTRACT

In this paper, we consider nonisothermal two-phase flows through heterogeneous porous media with periodic microstructure. Examples of such models appear in gas migration through engineered and geological barriers for a deep repository for radioactive waste, thermally enhanced oil recovery and geothermal systems. The mathematical model is given by a coupled system of two-phase flow equations, and an energy balance equation. The model consists of the usual equations derived from the mass conservation of both fluids along with the Darcy–Muskat and the capillary pressure laws. The problem is written in terms of the phase formulation, i.e. the saturation of one phase, the pressure of the second phase and the temperature are primary unknowns. The major difficulties related to this model are in the nonlinear degenerate structure of the equations, as well as in the coupling in the system. As fluid properties are defined as a function of temperature and pressure, there is a strong coupling between the mass balance and energy balance equations. Under some realistic assumptions on the data, we obtain a nonlinear homogenized coupled system of three coupled partial differential equations with effective coefficients (porosity, permeability, thermal conductivity, heat capacity) which are computed via solving cell problems. We give a rigorous mathematical derivation of the upscaled model by means of the two-scale convergence.

1. Introduction

Two-phase models for the simulation of flow and transport processes in the subsurface are used widely in various technical application fields. Among others, these applications include geothermal systems,

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oil reservoir engineering, ground-water hydrology, and thermal energy storage. More recently, modeling multiphase flow received an increasing attention in connection with gas migration in a nuclear waste repository and sequestration of CO_2.

This work aims to incorporate the temperature effects into immiscible incompressible two-phase flow in heterogeneous porous media with periodic microstructure. The system is subjected to significant changes in temperature conditions during long-term operation of the reservoir. Such a sophisticated mathematical description of the coupled processes is essential, taking into account nonisothermal two-phase flow. Modeling nonisothermal two-phase flow and transport processes in the subsurface requires the consideration of the transfer of energy between the phases in addition to the flow processes such as advection and diffusion. The basic equations for nonisothermal two-phase flow in a porous medium involve mass conservation, Darcy's law, energy conservation, saturation, and capillary pressure constraint equations. The description of the physical and thermodynamical state yields a system of three strongly coupled partial differential equations. The governing fluid and heat transport equations used to model thermal recovery processes are highly nonlinear. As fluid properties are defined as a function of temperature and pressure, there is a strong coupling between the mass balance and energy balance equations. The major difficulties related to this model are in the nonlinear degenerate structure of the equations, as well as in the coupling in the system.

In a previous paper [1], we gave an existence result of weak solutions for such a model under some realistic assumptions on the data. A model fully coupling the two-phase flow and heat transfer was developed to investigate immiscible incompressible two-phase flow in heterogeneous porous media under nonisothermal conditions. The goal of the present paper is to employ homogenization techniques to provide a rigorous derivation of an upscaled model by means of the two-scale convergence.

Over the past decades, mathematical analysis and numerical simulation of multiphase flows in porous media have been the subject of investigation of many researchers owing to important applications in reservoir simulation. There is an extensive literature on this subject. We will not attempt a literature review but will merely mention a few references. Here we restrict ourselves to the mathematical analysis of such models. We refer, for instance, to the books [2–8] and the references therein. The mathematical analysis and the homogenization of the system describing the flow of isothermal two incompressible immiscible fluids in porous media is quite understood. Existence, uniqueness of weak solutions to these equations, and their regularity has been shown under various assumptions on physical data; see for instance [2,3,5,9–14] and the references therein. There is a large and growing literature on homogenization techniques applied to multiphase flow in porous media. A recent review of the mathematical homogenization methods developed for incompressible single phase flow, incompressible immiscible two-phase flow in porous media and compressible miscible flow in porous media can be viewed in [7,15–20].

However, as reported in [1], all the aforementioned works are restricted to the case where flows are under isothermal conditions, contrarily to the present work. This assumption is too restrictive for some realistic problems, such as thermally enhanced oil recovery, geothermal energy production, high-level radioactive waste repositories. The present work was motivated by a need to incorporate the thermal behavior for such problems. In this work, a coupled reservoir two-phase flow model is described which accounts for varying reservoir temperature to capture flow physics accurately. Although considerable progress has been made in the computational simulation of two-phase problems under nonisothermal conditions (see e.g. [21–31] and the references therein), to the best knowledge of the authors, the homogenization of such coupled models under nonisothermal conditions is still missing. Closer to the present problem, recently homogenization for a Richard's model arising from the heat and moisture flow through a partially saturated porous medium was obtained in [32]. In [33], a model for nonisothermal single flow in double porosity media is constructed by the technique of homogenization.

This paper is concerned with the homogenization of a nonlinear degenerate system of diffusion–convection equations modeling the flow and transport of nonisothermal immiscible incompressible fluids through heterogeneous porous media, capillary and gravity effects being taken into account.