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# Volume viscosity and internal energy relaxation: Error estimates

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A B S T R A C T

We investigate the fast relaxation of internal energy in nonequilibrium gas models derived from the kinetic theory of gases. We establish uniform a priori estimates and existence theorems for symmetric hyperbolic–parabolic systems of partial differential equations with small second order terms and stiff sources. We prove local in time error estimates between the out of equilibrium solution and the one-temperature equilibrium fluid solution for well prepared data and *justify the apparition of volume viscosity terms.*

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## 1. Introduction

The kinetic theory of polyatomic gases shows that the volume viscosity coefficient is related to the time required for the internal and translational temperatures to come to equilibrium [1–7]. We establish in this paper local in time error estimates between the solution of an out of equilibrium two-temperature model and the solution of a one-temperature equilibrium model – including volume viscosity terms – when the relaxation time goes to zero.

The system of partial differential equations modeling fluids out of thermodynamic equilibrium as derived from the kinetic theory of gases is first summarized [5,6]. This system and its symmetrizability properties have been investigated in our previous work [8]. The symmetrizing normal variable $\mathsf{w}$ of the out of equilibrium model is taken in the form

$$\mathsf{w} = \Big(\rho, \boldsymbol{v}, \frac{1}{T_{\rm tr}} - \frac{1}{T_{\rm in}}, -\frac{1}{T}\Big)^t, \tag{1.1}$$

where $\rho$ denotes the gas density, $\boldsymbol{v}$ the fluid velocity, $T_{\rm tr}$ the translational temperature, $T_{\rm in}$ the internal temperature, and $T$ the local equilibrium temperature. The resulting system of partial differential equations

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is in the general form

$$\overline{\mathsf{A}}_0(\mathsf{w})\partial_t\mathsf{w} + \sum_{i\in\mathcal{D}}\overline{\mathsf{A}}_i(\mathsf{w})\partial_i\mathsf{w} - \epsilon_d\sum_{i,j\in\mathcal{D}}\partial_i\big(\overline{\mathsf{B}}_{ij}(\mathsf{w})\partial_j\mathsf{w}\big) + \frac{1}{\epsilon}\overline{\mathsf{L}}(\mathsf{w})\mathsf{w} = \epsilon_d\overline{\mathsf{b}}(\mathsf{w},\partial_{\mathsf{x}}\mathsf{w}_{\mathrm{II}}), \tag{1.2}$$

where $\partial_t$ denotes the time derivative operator, $\partial_i$ the space derivative operator in the $i$th direction, $\mathcal{D} = \{1,\dots,d\}$ the spatial directions, $d$ the space dimension, $\epsilon,\epsilon_d \in (0,1]$ two positive parameters and $\mathsf{w} = (\mathsf{w}_{\mathrm{I}},\mathsf{w}_{\mathrm{II}})^t$ is decomposed into its hyperbolic components $\mathsf{w}_{\mathrm{I}}$ and parabolic components $\mathsf{w}_{\mathrm{II}}$. The matrix $\overline{\mathsf{A}}_0$ is symmetric positive definite and bloc-diagonal, $\overline{\mathsf{A}}_i$ are symmetric, $\overline{\mathsf{B}}_{ij}^t = \overline{\mathsf{B}}_{ji}$, $\overline{\mathsf{B}}_{ij}$ have nonzero components only into the right lower $\overline{\mathsf{B}}_{ij}^{\mathrm{II,II}}$ blocs, $\overline{\mathsf{B}}^{\mathrm{II,II}} = \sum_{i,j\in\mathcal{D}}\overline{\mathsf{B}}_{ij}^{\mathrm{II,II}}(\mathsf{w})\xi_i\xi_j$ is positive definite for $\boldsymbol{\xi} \in \Sigma^{d-1}$, $\overline{\mathsf{L}}$ is positive semi-definite with a fixed nullspace $\overline{\mathcal{E}}$, and $\overline{\mathsf{b}}(\mathsf{w},\partial_{\mathsf{x}}\mathsf{w}_{\mathrm{II}})$ is quadratic in the gradients. Denoting by $\pi$ the orthogonal projector onto $\overline{\mathcal{E}}^{\perp}$, the normal variable $\mathsf{w}$ is such that we have the commutation relation $\pi\overline{\mathsf{A}}_0 = \overline{\mathsf{A}}_0\pi$. The source term is also naturally in quasilinear form as is typical in a relaxation framework and often encountered in mathematical physics [9]. The small parameter $\epsilon$ is associated with energy relaxation and the small parameter $\epsilon_d$ with second order dissipative terms.

We establish uniform a priori estimates for linearized symmetric hyperbolic–parabolic systems with small dissipation and stiff sources obtained from the nonlinear equations (1.2). Symmetrized forms are important for analyzing hyperbolic as well as hyperbolic–parabolic systems of partial differential equations modeling fluids [10–41]. A priori estimates are obtained uniformly with respect to the parameters $\epsilon_d \in (0,1]$ and $\epsilon \in (0,1]$. The differences with the estimates established by Kawashima [14] are the inclusion of extra terms associated with the fast variables $\pi\mathsf{w}/\epsilon$ and $\pi\mathsf{w}/\sqrt{\epsilon}$ as well as the coupling with the estimates for time derivatives. Denoting by $\mathsf{w}^\star$ a constant equilibrium state and $\bar{\tau}$ a positive time, we estimate $\mathsf{w}-\mathsf{w}^\star$ in the space $C^0\big([0,\bar{\tau}],H^l\big)$ as well as $\partial_t\mathsf{w}$ and $\pi\mathsf{w}/\epsilon$ in $L^2\big((0,\bar{\tau}),H^{l-1}\big)$ for $l \geq [d/2]+2$ where $H^l = H^l(\mathbb{R}^d)$ denotes the usual Sobolev space when the initial solution is close to the equilibrium manifold. A priori estimates require the commutation between the mass matrix and the orthogonal projector onto the fast manifold $\pi\overline{\mathsf{A}}_0 = \overline{\mathsf{A}}_0\pi$. These estimates lead to local existence theorems for well prepared initial conditions *on a time interval independent of both parameters* $\epsilon_d \in (0,1]$ and $\epsilon \in (0,1]$. Key points for local existence are notably to take into account stiff sources in the linearized equations in order to build approximated solutions, the new estimates for time derivatives, and the convergence rate of successive approximations that may depends on $\epsilon$. Stronger estimates for $\partial_t\mathsf{w}$ in $C^0\big([0,\bar{\tau}],H^{l-2}\big)$ as well as for $\pi\partial_t\mathsf{w}/\epsilon$ in $L^2\big((0,\bar{\tau}),H^{l-3}\big)$ with $l \geq [d/2]+4$ are also established when the initial time derivative is close to the equilibrium manifold. These theorems yield the first existence results for the out of equilibrium two-temperature model derived in [5] and symmetrized in [8]. On the other hand, the situation of ill prepared initial data lay beyond the scope of this work and we refer the reader to [42]. In the same vein, only local existence results are investigated and we refer to [43] for global existence results.

We finally investigate the singular limit $\epsilon,\epsilon_d \to 0$ in the system modeling fluids out of thermodynamic equilibrium. Various relaxation models have also been investigated in the literature in different physical and mathematical contexts [17,19,24,44,31,45,46,35,47,39,41]. In order to investigate the asymptotic behavior of solutions as $\epsilon,\epsilon_d \to 0$ we combine a priori estimates out of thermodynamic equilibrium with stability results associated with the equilibrium limit model. The fast variable notably corresponds to the rescaled temperature difference with $(T_{\mathrm{tr}}-T_{\mathrm{in}})/\epsilon = -T_{\mathrm{tr}}T_{\mathrm{in}}\pi\mathsf{w}/\epsilon$ and we use that perturbed hyperbolic–parabolic systems with small second order terms and perturbing right hand sides admit local solutions that depend continuously on perturbations. Denoting by $\mathsf{w}_{\mathrm{e}} = (\rho_{\mathrm{e}},\boldsymbol{v}_{\mathrm{e}},-1/T_{\mathrm{e}})^t$ the solution of the equilibrium one-temperature model including the volume viscosity terms and by $\varphi\mathsf{w} = (\rho,\boldsymbol{v},-1/T)^t$ the projection on the slow manifold of the normal variable $\mathsf{w}$ out of equilibrium, we establish that $\varphi\mathsf{w}-\mathsf{w}_{\mathrm{e}} = \mathcal{O}\big(\epsilon(\epsilon+\epsilon_d)\big)$. This justifies the addition of the volume viscosity term $-\kappa_{\mathrm{e}}(\boldsymbol{\nabla}\cdot\boldsymbol{v}_{\mathrm{e}})\boldsymbol{I}$ in the viscous tensor $\boldsymbol{\Pi}_{\mathrm{e}}$ at equilibrium

$$\boldsymbol{\Pi}_{\mathrm{e}} = -\kappa_{\mathrm{e}}(\boldsymbol{\nabla}\cdot\boldsymbol{v}_{\mathrm{e}})\boldsymbol{I} - \eta_{\mathrm{e}}\big(\boldsymbol{\nabla}\boldsymbol{v}_{\mathrm{e}} + (\boldsymbol{\nabla}\boldsymbol{v}_{\mathrm{e}})^t - \frac{2}{3}(\boldsymbol{\nabla}\cdot\boldsymbol{v}_{\mathrm{e}})\boldsymbol{I}\big),$$

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