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The Cauchy problem for quadratic and cubic Ostrovsky equation with negative dispersion

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ABSTRACT

In this paper, we consider the Cauchy problem for the quadratic and cubic Ostrovsky equation with negative dispersion

$$\partial_x\left(u_t-\beta\partial_x^3u+\frac{1}{k}\partial_x(u^k)\right)-\gamma u=0,\beta<0,\gamma>0,(k=2,3).$$

Firstly, by using the Strichartz estimates instead of the Cauchy–Schwarz inequalities, we give an alternative proof of Lemma 1.2 of Isaza and Mejía (2006). Secondly, by using the Strichartz estimates instead of the Cauchy–Schwarz inequalities, we give an alternative proof of Lemma 1.3 of Isaza and Mejía (2007). Thirdly, we prove that the Cauchy problem for the cubic Ostrovsky equation is locally well-posed in $H^s(\mathbf{R})$ with $s\geq\frac{1}{4}$. Finally, we prove that the Cauchy problem for the cubic Ostrovsky equation is not well-posed in $H^s(\mathbf{R})$ with $s<\frac{1}{4}$.

1. Introduction

In this paper, we are concerned with the Cauchy problem for quadratic and cubic Ostrovsky equation with negative dispersion

$$\partial_x\left(u_t-\beta\partial_x^3u+\frac{1}{k}\partial_x(u^k)\right)-\gamma u=0,\gamma>0,\beta<0,k=2,3, \tag{1.1}$$

$$u(x,0)=u_0(x). \tag{1.2}$$

Here $u(x,t)$ represents the free surface of the liquid and the parameter $\gamma>0$ measures the effect of rotation. (1.1) describes the propagation of internal waves of even modes in the ocean, for instance, see the works of Galkin and Stepanyants [1], Leonov [2], and Shrira [3,4] and Ostrovskii [5]. The parameter β determines the

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type of dispersion, more precisely, $\beta < 0$ (negative dispersion) for surface and internal waves in the ocean or surface waves in a shallow channel with an uneven bottom and $\beta > 0$ (positive dispersion) for capillary waves on the surface of liquid or for oblique magneto-acoustic waves in plasma [1,6–8].

Now we give a brief review about the quadratic Ostrovsky equation. Some authors have investigated the stability of the solitary waves or soliton solutions to (1.1), for instance, see [9–15]. Varlamov and Liu [16] studied the solitary waves and fundamental solution for Ostrovsky equation. Many people have studied the Cauchy problem for (1.1), for instance, see [10,12,14,17–31]. Isaza and Mejía [22] and Tsugawa [32] showed that the Cauchy problem for (1.1) is locally well-posed in $H^s(\mathbf{R})$ with $s > -\frac{3}{4}$. Isaza and Mejía [24] proved that the Cauchy problem for (1.1) is not quantitatively well-posed in $H^s(\mathbf{R})$ with $s < -\frac{3}{4}$. Coclite and Ruvo [33,34] have proved that as the diffusion parameter tends to zero, the solutions to (1.1) with $k = 2$ converge to discontinuous weak solutions of the Ostrovsky–Hunter equation and the asymptotic behavior as γ tends to zero. Recently, by using the modified Besov spaces, Li et al. [35] proved that the Cauchy problem for (1.1) with $k = 2$ is locally well-posed in $H^{-\frac{3}{4}}(\mathbf{R})$. Very recently, by paying more attention to the structure of (1.1) with $k = 2$, Yan et al. [36] established the local well-posedness of the Ostrovsky equation in $H^{-\frac{3}{4}}(\mathbf{R})$ with positive dispersion.

Levandosky and Liu [9] investigated the stability of solitary waves of the generalized Ostrovsky equation

$$[u_t - \beta u_{xxx} + (f(u))_x]_x = \gamma u, x \in \mathbf{R}, \tag{1.3}$$

where f is a C^2 function which is homogeneous of degree $p \geq 2$ in the sense that it satisfies $sf'(s) = f(s)$. Levandosky [15] investigated the stability of ground state solitary waves of (1.3) with homogeneous nonlinearities of the form $f(u) = c_1|u|^p + c_2|u|^{p-1}u$, $c_1, c_2 \in \mathbf{R}$.

Motivated by [37–40], firstly, by using the $X_{s,b}$ spaces defined in page 5 and the Strichartz estimates instead of the Cauchy–Schwarz inequalities, we give an alternative proof of Lemma 1.2 of [22]. Secondly, by using the $X_{s,b}$ spaces defined in page 5 and the Strichartz estimates instead of the Cauchy–Schwarz inequalities, we give an alternative proof of Lemma 1.3 of [23]. Thirdly, by using the $X_{s,b}$ spaces defined in page 5 and the Strichartz estimates, we prove that (1.1) with $k = 3$ is locally well-posed in $H^s(\mathbf{R})$ with $s \geq \frac{1}{4}$; finally, we prove that the problem (1.1) with $k = 3$ is not well-posed in $H^s(\mathbf{R})$ with $s < \frac{1}{4}$.

If u is the solution to (1.1), then $v(x,t) = \beta^{-1}u(x, \beta^{-1}t)$ is the solution $v_t - v_{xxx} + \frac{1}{2}\partial_x(v^2) - \beta^{-1}\gamma\partial_x^{-1}v = 0$. If u is the solution to (1.1), then $v(x,t) = \beta^{-\frac{1}{2}}u(x, \beta^{-1}t)$ is the solution $v_t - v_{xxx} + \frac{1}{3}\partial_x(v^3) - \beta^{-1}\gamma\partial_x^{-1}v = 0$. Hence without loss of generality, we can always assume that $\gamma = -\beta = 1$ in this paper.

We introduce some notations before giving the main result. Throughout this paper, we assume that C is a positive constant which may vary from line to line and $0 < \epsilon < 10^{-4}$. $a \sim b$ means that $|b| \leq |a| \leq 4|b|$. $a \gg b$ means that $|a| > 4|b|$. $\psi(t)$ is a smooth function supported in $[0,2]$ and equals 1 in $[0,1]$. We assume that $\mathscr{F}u$ is the Fourier transformation of u with respect to both space and time variables and $\mathscr{F}^{-1}u$ is the inverse transformation of u with respect to both space and time variables, while $\mathscr{F}_x u$ denotes the Fourier transformation of u with respect to the space variable and $\mathscr{F}_x^{-1}u$ denotes the inverse transformation of u with respect to the space variable. Let

$$\langle \cdot \rangle = 1 + |\cdot|, \phi(\xi) = \xi^3 - \frac{1}{\xi}, \phi_j(\xi) = \xi_j^3 - \frac{1}{\xi_j},$$
$$\sigma = \tau - \phi(\xi), \sigma_j = \tau_j - \phi(\xi_j)(j = 1, 2).$$

Space $X_{s,b}$ is defined by

$$X_{s,b} = \left\{ u \in \mathscr{S}'(\mathbf{R}^2) : \|u\|_{X_{s,b}} = \left\| \langle \xi \rangle^s \langle \sigma \rangle^b \mathscr{F}u(\xi, \tau) \right\|_{L^2_{\tau\xi}(\mathbf{R}^2)} < \infty \right\}.$$