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Global stability of large solutions for the Navier–Stokes equations with Navier boundary conditions

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A B S T R A C T

We prove global stability of strong large solutions for the 3D incompressible Navier–Stokes equations with Navier slip boundary conditions. This result is obtained under an integrable property that depends on these slip conditions. In addition, we show that strong helical solutions are global in time. Combining the latter with the stability result, we provide a class of 3D global large solutions with perturbations of helical vector-fields as initial data.

1. Introduction

We consider the 3D incompressible Navier–Stokes equations

$$\begin{cases} \partial_t u + [u\cdot\nabla]u - \mu\Delta u + \nabla p &= f \quad \text{in} \quad (x,t)\in\Omega\times[0,T), \\ div\, u &= 0 \quad \text{in} \quad (x,t)\in\Omega\times[0,T), \end{cases} \tag{1.1}$$

where the unknowns are the velocity field $u = (u_1(x,t), u_2(x,t), u_3(x,t))$ and the scalar pressure $p = p(x,t)$, and the convection term $[u\cdot\nabla]u$ is given by $\sum_{i=1}^{3} u_i\partial_{x_i}u$. The constant $\mu > 0$ and vector-field $f = f(x,t)$ stand for the viscosity and the external force acting on the fluid, respectively. If Ω is bounded, a boundary condition widely used is the no-slip condition

$$u_\Gamma = 0, \tag{1.2}$$

where Γ is the boundary of Ω.

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There is an extensive literature related to the existence, uniqueness and regularity theory of solutions for the system (1.1) with boundary condition (1.2). See [1] for an overview about this subject. Roughly speaking, there exist global weak solutions and local strong solutions (with uniqueness). Global existence of strong solutions is obtained for small initial data. Uniqueness of weak solutions and global existence of strong large solutions are classical open problems. Global strong large solutions are secured for a few symmetrical flows such as two-dimensional [2], axial with no swirl [3] and helical [4].

In [5], Ponce et al. showed that there exists a family of 3D global large solutions for (1.1)–(1.2) by proving stability around symmetric solutions as well as stability around other global strong solutions that have a suitable integrable property. Similar results can be found in [6–13] and [14]. This type of result has been extended to the non-homogeneous case [15], Boussinesq system [16,17], 2D dissipative quasi-geostrophic flows [18], MHD and Hall-MHD systems [19,20]. We point out that none of them have considered interactions involving sliding along the border.

Although (1.2) is commonly used, many experiments indicate different types of interaction in the boundary (see [21] and [22]). In this paper, we consider the Navier slip conditions

$$\left\{\begin{array}{rcl} u.\eta|_{\Gamma} & = & 0, \\ (\alpha u + [D[u]\eta]_{tg})|_{\Gamma} & = & 0. \end{array}\right. \tag{1.3}$$

Here the vector-field $\eta = \eta(x)$ is the unity normal exterior to the boundary, α is a fixed positive friction coefficient, $D[u]$ is the 3×3 matrix with entries

$$d_{i,j}[u] = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \tag{1.4}$$

and $[\,v\,]_{tg}|_{\Gamma}$ is the tangential component of v with respect to Γ (namely, $[\,v\,]_{tg}|_{\Gamma} = \eta \times v \times \eta$). The first equality in (1.3) is the condition of impermeability, while the second means that the tangential component of the stress tensor is proportional to the velocity.

Global weak solutions as well as local strong solutions (with uniqueness) for the Navier–Stokes equation with the condition (1.3) can be found in [23–26] and [27]. Uniqueness of 2D weak solutions and regularity are proved in [28,29] and [30]. In [31] a stability result is proved with another type of slip boundary condition and in the L^3-context.

First, in spirit of [5], we prove that, for each global large strong solution of (1.1)–(1.3) satisfying a suitable integrable property, there exists a neighborhood around it where all strong solutions are likewise global in time (see Section 2). The integrable requirement (2.13) depends on the Navier slip conditions via the norm $\|\cdot\|_{\mathcal{S}(\Omega)}$ (see (2.2)). Considering periodic channel, we show that solutions of (1.1)–(1.3) preserve helical symmetry in the sense that if a smooth initial data is helical, then the solution remains helical along time. After, these solutions are showed to be global in time and stable (see Section 3). The stability result also includes perturbations of 2D solutions. In fact, in these two cases, the integrable hypothesis can be removed. So, we provide two classes of genuinely 3D global large strong solutions (not necessarily symmetric) corresponding to initial data being perturbations of either helical or 2D vector-fields.

The outline of this paper is as follows. In Section 2, we deal with 3D smooth bounded domains. First, we give some estimates and the notion of strong solution for the problem. After, we state and prove our results. Section 3 is devoted to infinite channel domains, global extension in time for helical solutions, and stability. The main results in this section are proved in Section 3.3.

2. 3D bounded domains

2.1. Basic definitions and estimates

Let us state some definitions and basic properties that will be used in this work. For now we suppose Ω a 3D smooth bounded domain. The usual vector Sobolev spaces are denoted by $H^m(\Omega)$ and the Leray