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Decay estimate of solutions to the coupled chemotaxis–fluid equations in $\mathbb{R}^3$

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ABSTRACT

We are concerned with a model arising from biology, which is coupled system of the chemotaxis equations and the viscous incompressible fluid equations through transport and external forcing. We study the large time behavior of solutions near a constant states to the chemotaxis-Navier–Stokes system in $\mathbb{R}^3$. Appealing to a pure energy method, we first obtain a global existence theorem by assuming that the H^3 norm of the initial data is small, but the higher order derivatives can be arbitrary large. If the initial data belongs to homogeneous Sobolev norms $\dot{H}^{-s}$ $(0 \le s < \frac{3}{2})$ or homogeneous Besov norms $\dot{B}^{-s}_{2,\infty}$ $(0 < s \le \frac{3}{2})$, we obtain the optimal decay rates of the solutions and its higher order derivatives. As an immediate byproduct, we also obtain the usual $L^p - L^2$ $(1 \le p \le 2)$ type of the decay rates without requiring that the L^p norm of initial data is small.

1. Introduction

We consider a mathematical model for the motion of swimming bacteria in an incompressible viscous fluid. As described in the pioneering literature Keller–Segel [1], chemotaxis as a biological process responsible for some instances of such demeanor, which is directed movement of living cells (e.g. bacteria) that move toward a chemically more favorable environment under the effects of chemical gradients. We shall note that when no chemical are present, the movement of cells is completely random, when a attractant chemical is present, the motility changes, the tumbles become less frequent so that the cells move toward the chemical attractant. This is important for microorganism to find food (e.g. glucose) by swimming toward the highest concentration of food molecules. A large quantity of mathematical model has been developed, see [2–4] for instance. Furthermore, in [5] it can be observed experimentally that bacteria are suspend in the fluid, which is influenced by the gravitational forcing generated by the aggregation of cells. Moreover, the oxygen plays an important role in the reproduce of aerobic bacteria. For example, Bacillus subtilis often live in thin fluid

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layers near solid–air–water contact line, in which the swimming bacteria move toward higher concentration of oxygen according to mechanism of chemotaxis. We note further that both the oxygen concentration, chemical attractant and bacteria density are transported by the fluid and diffuse through the fluid [5–8]. In present paper, we deal with the chemotaxis-Navier–Stokes system in $\mathbb{R}^3$:

$$\begin{cases} \partial_t n + u\cdot\nabla n = \Delta n - \nabla\cdot(\chi(c)n\nabla c) - \nabla\cdot(\tau(v)n\nabla v) + h(n), \\ \partial_t c + u\cdot\nabla c = \Delta c - f(c)n, \\ \partial_t v + u\cdot\nabla v = \Delta v - \gamma v + g(v)n, \\ \partial_t u + u\cdot\nabla u = \Delta u - \nabla P - n\nabla\phi, \\ \nabla\cdot u = 0, \quad t>0, \quad x\in\mathbb{R}^3, \end{cases} \tag{1.1}$$

with initial data

$$(n,c,v,u)|_{t=0} = (n_0(x), c_0(x), v_0(x), u_0(x)) \quad \text{in } \mathbb{R}^3, \tag{1.2}$$

and it is supposed to hold that

$$(n_0(x), c_0(x), v_0(x), u_0(x)) \longrightarrow (n_\infty, 0, 0, 0) \quad \text{as } |x|\longrightarrow\infty, \tag{1.3}$$

where n_∞ is a positive constant. Here, the unknowns are $n = n(t,x) : \mathbb{R}^+\times\mathbb{R}^3 \longrightarrow \mathbb{R}^+, c = c(t,x) : \mathbb{R}^+\times\mathbb{R}^3 \longrightarrow \mathbb{R}^+, v = v(t,x) : \mathbb{R}^+\times\mathbb{R}^3 \longrightarrow \mathbb{R}^+, u = u(t,x) : \mathbb{R}^+\times\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ and $P = P(t,x) : \mathbb{R}^+\times\mathbb{R}^3 \longrightarrow \mathbb{R}$ denote the cell density, oxygen concentration, chemical attractant concentration, fluid velocity field and the pressure of the fluid, respectively; while $n_0 = n_0(x), c_0 = c_0(x), v_0 = v_0(x)$ and $u_0 = u_0(x)$ are the given functions. $\chi(c)$ and $\tau(v)$ are the chemotactic sensitivity, $f(c)$ denote the consumption rate of oxygen, $h(n) = \alpha_1 n - \alpha_2 n^2$ with $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, $g(v)$ is the consumption rate of chemical attractant by the cells, $\phi = \phi(t,x)$ is a given potential function, the constant $\gamma > 1$.

We call (1.1)–(1.3) as the double chemotaxis model under the effect of the Navier–Stokes fluid (cf. [9]), such a model had been first introduced by Tuval et al. [10]. Here, we present some relate work about chemotaxis–fluid. For the system (1.1)–(1.2) with four unknowns $\{n, c, u, P\}$ and $\alpha_1 = \alpha_2 = 0$:

$$\begin{cases} \partial_t n + u\cdot\nabla n = \Delta n - \nabla\cdot(\chi(c)n\nabla c), \\ \partial_t c + u\cdot\nabla c = \Delta c - f(c)n, \\ \partial_t u + u\cdot\nabla u = \Delta u - \nabla P - n\nabla\phi, \\ \nabla\cdot u = 0, \end{cases} \tag{1.4}$$

which is so-called a single chemotaxis, and has been widely studied in the last few years. In particular, the consumption rate of oxygen $f(c)$ is a step function in [10]. Since the difficulties associated with the Navier–Stokes equations in three dimensional domains, many of these works focus on more favorable variants of the model, for example by resorting to the Stokes equation upon neglect of the nonlinear convective term [11,12] or on the two dimensional case [11,13–16]. Under the assumption that $\chi(c) = C$ is a constant and $f(\cdot)$ is monotonically increasing with $f(0) = 0$, Lorz [7] constructed a local solutions to (1.4) in a bounded domain $\Omega \subset \mathbb{R}^N$, $(N = 2, 3)$ with no-flux boundary condition and in $\mathbb{R}^2$ in the case of homogeneous Dirichlet conditions for oxygen. In whole space $\mathbb{R}^N$ $(N = 2, 3)$, Duan et al. [17] obtained global existence and rates of convergence on classical solutions near constant states in $\mathbb{R}^3$ under the assumption initial data small enough, and further obtained the weak solution in $\mathbb{R}^2$ under smallness assumption on either $\nabla\phi$ or the initial data for oxygen concentration. Winkler [11] obtained the existence of global weak solution to (1.4) with large initial data in bounded convex domain $\Omega \subset \mathbb{R}^3$ with smooth boundary, see also [18] for even milder diffusion effects. If both $\chi(\cdot)$ and $f(\cdot)$ are suppose to be nonnegative and nondecreasing, Chae et al. [19] shown that the Cauchy problem of (1.4) admits a global classical solution under the assumption that $\sup_c|\chi(c) - \mu f(c)|$ be sufficiently small for some $\mu > 0$. By the same author in [20] obtained the global existence of smooth solutions as well as temporal decay. With a concept of eventual energy solution, Winkler [21] showed that