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Global strong solution for initial–boundary value problem of one-dimensional compressible micropolar fluids with density dependent viscosity and temperature dependent heat conductivity

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we prove existence and uniqueness of global strong solution of the compressible micropolar fluids model in one dimensional space with density dependent viscosity and temperature dependent heat conductivity under stress-free and thermally insulated boundary conditions. Former studies regard the coefficients as constants while we consider that the viscosity depends on density and the heat conductivity depends on temperature, which lead to the high nonlinearity and difficulty on deducing the bounds of both density and temperature.

1. Introduction

The model of micropolar fluids which respond to micro-rotational motions and spin inertia was first introduced by Eringen [1] in 1966. This model is more proper than Navier–Stokes equations to describe the motions of a large variety of complex fluids consisting of dipole elements such as the suspensions, animal blood, and liquid crystal. For more physical background, please refer to [2,3]. In Euler coordinates, it was formulated in [4] as follows:

$$\begin{cases} \dot{\rho} + \rho div\mathbf{u} = 0, \\ \rho\dot{\mathbf{u}} = div\mathbf{T} + \rho\mathbf{f}, \\ \rho j\dot{\omega} = div\mathbf{C} + \mathbf{T}_x + \rho\mathbf{g}, \\ \rho\dot{e} = \mathbf{T} : \nabla\mathbf{u} + C : \nabla\omega - \mathbf{T}_x \cdot \omega + div\mathbf{q} + \rho\delta, \end{cases} \tag{1.1}$$

where $\dot{\alpha}$ denotes material derivative of a field α:

$$\dot{\alpha} = \frac{\partial\alpha}{\partial t} + (\nabla\alpha)\cdot\mathbf{u}, \tag{1.2}$$

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with notation:

$$\begin{aligned}
&\rho\text{—mass density}\\
&\mathbf{u}\text{—velocity}\\
&\text{sym}\mathbf{T}=\frac{1}{2}(\mathbf{T}+\mathbf{T}^T),\ \text{skw}\mathbf{T}=\frac{1}{2}(\mathbf{T}-\mathbf{T}^T)\\
&p\text{—pressure}\\
&\mathbf{T}\text{—stress tensor}\\
&\mathbf{T}_x\text{—an axial vector with the cartesian components}(\mathbf{T}_x)_i=\epsilon_{ijk}T_{kj},\text{where}\\
&\epsilon_{ijk}\text{ is the alternating tensor}\\
&\omega\text{—microrotation velocity}\\
&\omega_{\text{skw}}\text{—a skew tensor with the Cartesian components}(\omega_{\text{skw}})_{ij}=\epsilon_{ijk}\omega_k\\
&j\text{—microinertia density (a positive scalar field)}\\
&\mathbf{C}\text{—couple stress tensor}\\
&\theta\text{—absolute temperature}\\
&e\text{—internal energy density}\\
&\mathbf{q}\text{—heat flux density vector}\\
&\mathbf{f}\text{—body force density}\\
&\mathbf{g}\text{—body couple density}\\
&\delta\text{—body heat density.}
\end{aligned}\tag{1.3}$$

The linear constitutive equations for stress tensor, couple stress tensor and heat flux density vector are, respectively, of the forms:

$$\begin{aligned}
&\mathbf{T}=(-p+\lambda div\mathbf{u})I+2\mu\ \text{sym}\nabla u-2\mu_r\ (\text{skw}\nabla\mathbf{u}+\omega_{\text{skw}}),\\
&\mathbf{C}=(c_0 div\omega)I+2c_d\ \text{sym}\nabla\ \omega-2c_a\ \text{skw}\nabla\omega,\\
&\mathbf{q}=-\kappa\nabla\theta,
\end{aligned}\tag{1.4}$$

where λ and μ are coefficients of viscosity and $\mu_r,\ c_0,\ c_a$ and c_d are the coefficients of microviscosity. Because of the Clausius–Duhamel inequalities they must have the properties:

$$\mu,\ \mu_r,\ c_d\geq 0,\quad \lambda+2\mu\geq 0,\quad c_0+2c_d\geq 0.\tag{1.5}$$

By the constant $\kappa(\kappa\geq 0)$ we denote the heat-conduction coefficient.

Much attention has been paid to this model by many mathematicians among which Mujaković initialed the study on this model for compressible flow. First, for one-dimensional compressible flow, she made a series of efforts in studying the local-in-time existence and uniqueness, the global existence and regularity of solutions to an initial–boundary value problem with both homogeneous [5–7] and non-homogeneous [8–11] boundary conditions respectively. Besides, she also analyzed large time behavior of the solutions and the stabilization of solutions to the Cauchy problem [12–14]. Second, for three-dimensional model, Mujaković and her collaborator Dražić studied the local existence, global existence, uniqueness and large time behavior of spherical symmetry solutions [15,4,16–18]. There are other authors who contribute to the study of this model such as Chen [19] who proved the global existence of strong solutions to the Cauchy problem with initial vacuum for one-dimensional model. Huang and Nie [20] obtained exponential stability with homogeneous boundary conditions. There are other papers [21–23] which studied the local stability of rarefaction wave, contact wave as well as viscous shock wave. For the three-dimensional model, Chen [24], Chen, Huang and Zhang [25] proved a blow up criterion of strong solutions to the Cauchy problem. Chen, Xu and Zhang [26] established the global weak solutions with discontinuous initial data and vacuum. Recently, Liu and Zhang [27] have obtained the optimal time decay of the three-dimensional compressible flow. Huang