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Global existence and asymptotic behavior to a chemotaxis–consumption system with singular sensitivity and logistic source☆

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ABSTRACT

We consider a chemotaxis–consumption system with singular sensitivity and logistic source: $u_t = \Delta u - \nabla \cdot (u\phi(v)\nabla v) + ru - \mu u^k$, $v_t = \Delta v - uv$ in a smooth bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$, where $r, \mu > 0$, $k > 1$, and $\phi(s) \in C^1(0, \infty)$ satisfying $\phi(s) \to \infty$ as $s \to 0$. It is proved that there exists a global classical solution if $k > 1$ for $n = 1$ or $k > 1 + \frac{n}{2}$ for $n \geq 2$. The asymptotic behavior of solutions is determined as well for $\phi(v) = \frac{1}{v}$, $n = 2$ that if $k > 2$, there exists $\mu_* > 0$ such that $(u, v, \frac{|\nabla v|}{v}) \to ((\frac{r}{\mu})^{\frac{1}{k-1}}, 0, 0)$ as $t \to \infty$ provided $\mu > \mu_*$.

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1. Introduction

Chemotaxis, the directed movement of cells or bacteria to the external chemical signal, plays a significant role in a wide range of biological processes. The motile Escherichia coli (cells or bacteria), when placed in one end of a capillary tube containing oxygen and an energy source (chemical signal), will form visible bands and move away from this end at constant speed [1]. Keller and Segel proposed in [2] a phenomenological model to capture this kind of behavior, described via

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u\phi(v)\nabla v), \\ v_t = \epsilon\Delta v - uv, \end{cases} \tag{1.1}$$

where $\epsilon > 0$ and $\phi(v) = \frac{\chi}{v^\alpha}$ with $\alpha \geq 1$. Such visible bands are a consequence of a chemotaxis–consumption process, i.e., the bacteria u have the ability to partially orient their movement toward higher concentration gradient of oxygen v, which is consumed by u as a nutrient through contacting. It is mentioned that the

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particular choice of the sensitivity function $\phi(v) = \frac{\chi}{v^\alpha}$ selected is due to the Weber–Fechner's law of stimulus perception in the process of chemotactic response. Intuitively, the difficulties of studying the global existence and further global dynamic behavior of solutions to (1.1) come from the interplay of the consumptive effect with the singular chemotactic mechanism: the oxygen v shrinks via the second equation $(1.1)_2$ during evolution, and then enhances the chemotactic strength of the bacteria because of the singular behavior when $v \approx 0$ in $(1.1)_1$. On the other hand, this delicate absorption-taxis interplay may generate wave-like solutions [2].

Contrary to the second equation $(1.1)_2$ with the oxygen consumed by the bacteria as a nutrient, the original model proposed by Keller and Segel [3] involves a chemical signal actively secreted by the bacteria themselves,

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (\frac{u}{v} \nabla v) + f(u) & x \in \Omega, \quad t > 0, \\ \tau v_t = \Delta v - v + u, & x \in \Omega, \quad t > 0, \end{cases} \tag{1.2}$$

where $\chi > 0$, $\tau \in [0, 1]$, $f(u) \in C^1(0, \infty)$ with $f(0) \geq 0$. For $f(u) \equiv 0$ and $\tau = 0$, it is known that all radial classical solutions are global-in-time if either $n = 2$ with $\chi > 0$ arbitrary, or $n \geq 3$ with $\chi < \frac{2}{n-2}$ [4]. If $\chi < \frac{2}{n}$ with $n \geq 1$, there exists a unique globally bounded classical solution [5]. For $n = 2$, it was shown in fact that (1.2) admits even classical non-radial solutions globally bounded for arbitrary $\chi > 0$ [6]. When $f(u) \equiv 0$ and $\tau = 1$, the unique solution of (1.2) is global [7] and globally bounded [8] if $\chi < \sqrt{\frac{2}{n}}$. Moreover, there exists a global bounded classical radial solution for arbitrary $\chi > 0$ when $\tau > 0$ sufficiently small in a bounded radial domain $\Omega \subset \mathbb{R}^2$ [9]. For $\tau = 0$ and $f(u) = ru - \mu u^2$ with $r \in \mathbb{R}$ and $\mu > 0$, it has been proved that the system (1.2) exists a global bounded classical solution in [10] if $r > \frac{\chi^2}{4}$ for $0 < \chi \leq 2$, or $r > \chi - 1$ for $\chi > 2$. Currently, we proved that the same is true with $\tau = 1$ and the $f(u)$ mentioned above [11].

Now, recall a more general chemotaxis–consumption system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u\phi(v)\nabla v) + f(u), & x \in \Omega, \quad t > 0, \\ v_t = \Delta v - uv, & x \in \Omega, \quad t > 0, \end{cases} \tag{1.3}$$

where $\phi(s) \in C^1(0, \infty)$ and $f(s) \in C^0[0, \infty)$. For the case of $\phi(v) = \chi$ and $f(u) \equiv 0$, it was established that (1.3) with $\chi = 1$ possesses a unique global bounded classical solution for $n = 2$ or an asymptotically smooth weak solution for $n = 3$ [12]. Globally bounded classical solutions of (1.3) were established if $0 < \chi \leq \frac{1}{6(n+1)\|v_0\|_{L^\infty(\Omega)}}$ [13]. It was obtained for $\phi(v) = \chi$ and $f(u) = ku - \mu u^2$ also that (1.3) admits a globally bounded classical solution provided μ suitably large, or just a global weak solution if $\mu > 0$ [14]. We refer to [15–18] for more results on chemotaxis–consumption systems with nonlinear diffusion or rotational flux terms. For the special case of $\phi(v) = \frac{\chi}{v}$ and $f(u) = 0$ with $n = 2$, there exists a global generalized solution to (1.3) with $v \to 0$ in $L^p(\Omega)$ as $t \to \infty$ [19], and the solution becomes eventually smooth and converges to the homogeneous steady state if the initial mass $\int_\Omega u_0 dx$ is small [20]. In particular, under an explicit smallness condition on $u_0 \ln u_0 \in L^1(\Omega)$ and $\nabla \ln v_0 \in L^2(\Omega)$, the system (1.3) possesses a global classical solution [20]. Moreover, in a radially symmetric setting, a "normalized solution" has been constructed for $n \geq 3$ [21]. In addition, for the chemotaxis–consumption with nonlinear diffusion and singular sensitivity, i.e.,

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u - \frac{u}{v}\nabla v), & x \in \Omega, \quad t > 0, \\ v_t = \Delta v - uv, & x \in \Omega, \quad t > 0, \end{cases} \tag{1.4}$$

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