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Global solutions to compressible Navier–Stokes equations with spherical symmetry and free boundary

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A B S T R A C T

This work is devoted to study the global existence of strong and classical solutions to the compressible Navier–Stokes equations with or without a density jump on the moving boundary for the spherically symmetric motion. We establish a unified method to track the propagation of regularity of strong and classical solutions which works for the cases when the density connects to vacuum continuously and with a jump simultaneously. The result we obtain is able to deal with both strong solutions with physical vacuum for which the sound speed is 1/2-Hölder continuous across the boundary, and classical solutions with physical vacuum when $1 < \gamma < 3$. In contrast to the previous results of global weak solutions, we track the regularity globally-in-time up to the symmetry centre and the moving boundary. In particular, the free boundary can be traced.

1. Introduction

1.1. Description and background

The motion of a viscous barotropic gas (or fluid) can be described by the isentropic compressible Navier–Stokes equations. In particular, the following system with constant viscosities governs the spherical motion in three dimensional space,

$$\begin{cases} \partial_t(r^2\rho) + \partial_r(r^2\rho u) = 0 & r \in (0, R(t)), \\ \partial_t(r^2\rho u) + \partial_r(r^2\rho u^2) + r^2\partial_r P = (2\mu+\lambda)r^2\partial_r\left(\dfrac{\partial_r(r^2 u)}{r^2}\right) & r \in (0, R(t)), \end{cases} \tag{1.1}$$

where $\rho, u, R(t)$ represent the density, the radial velocity and the radius of the boundary respectively. The Lamé constants μ, λ denoting the viscosity coefficients satisfy the relations $\mu > 0, 3\lambda + 2\mu \geq 0$. Moreover,

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the pressure potential P is assumed to depend only on the density. For simplicity in this work, the equation of state is taken as $P = \rho^\gamma$ with $\gamma > 1$. Also, we will work on the Navier–Stokes system (1.1) complemented with the following free boundary conditions,

$$\begin{gathered}[P - (2\mu r^2 \partial_r u + \lambda \partial_r (r^2 u))](R(t), t) = 0, \\ u(0,t) = 0,\ \partial_t R(t) = u(R(t), t),\end{gathered} \tag{1.2}$$

where the first boundary condition represents the balance of stress tensor across the gas–vacuum interface. Also, the initial data is taken to be

$$R(0) = R_0,\ u(r,0) = u_0(r),\ \rho(r,0) = \rho_0(r),\ r \in (0, R_0). \tag{1.3}$$

Without loss of generality, it is assumed $R_0 = 1$ in the following. Meanwhile, we do not impose any condition on the boundary profile of the initial density ρ_0. In fact, ρ_0 can connect to the vacuum with or without a jump.

In particular, ρ_0 can approach the vacuum continuously across the boundary. Such a gas–vacuum interface problem has appeared in plenty of physical scenarios such as astrophysics, shallow water waves, etc. For example, the configuration of a non-rotating gaseous star admits the physical vacuum boundary, i.e.

$$-\infty < \nabla_n c^2 \leq -C < 0, \rho = 0, \qquad \text{on the boundary}, \tag{1.4}$$

where n denotes the normal direction and $c^2 = P'(\rho)$ is the square of the sound speed. Indeed, the physical vacuum boundary indicates that the sound speed c is only 1/2-Hölder continuous instead of Lipschitz continuous across the boundary, which is quite troublesome (see [1]). Only recently, some local-in-time well-posedness of the smooth solutions for such problems is available for the inviscid flows [2–9] with or without self-gravitation and for the viscous flows [10] with self-gravitation. As for the global dynamic of flows with the physical vacuum boundary (1.4), by studying the nonlinear stability of some self-similar solutions, Hadžić and Jang show that there exist global large solutions to the Euler equations and Euler–Poisson equations as a perturbation of the expanding homogeneous solutions [11,12]. Also, Luo and Zeng [13,14] study the asymptotic behaviour of Euler equations with damping and show that the solutions converge to the Barenblatt self-similar solutions for the porous media equations in one-dimensional and spherically symmetric setting. On the other hand, for the viscous flows, Luo, Xin, Zeng [15] have shown that with a small perturbation of the Lane–Emden solutions, the strong solution to the Navier–Stokes–Poisson system exists globally and converges to the equilibrium state. See [16] for the case with degenerate viscosities. Meanwhile, Zeng has established the global regularity of the compressible Navier–Stokes equations in the one dimensional setting which includes the case of physical vacuum in [17]. Zeng's work extends the one in [18], in which the authors have shown the global existence of the solutions to the one dimensional problem with constant viscosities but higher regularity for the density on the boundary.

When the density connects to the vacuum with a jump, a global weak solution with a spherically symmetric motion to the problem with density-dependent viscosities is obtained in [19] by Guo, Li, Xin. Moreover the solution is shown to be smooth away from the centre. Recently, such a problem is studied in the setting of spherical symmetry in two dimensional space and $\mu = constant$, $\lambda = \lambda(\rho) = \rho^\beta$ with some $\beta > 1$ by Li, Zhang [20]. Working in both Lagrangian and Eulerian coordinates, the authors show the global existence of strong solutions. The regularity obtained by Li and Zhang is in terms of the velocity vector field $U = u \cdot \vec{x}/r$ where $r = |\vec{x}|, \vec{x} \in \mathbb{R}^2$ rather than on u and therefore has not covered the result achieved here. Another noticeable result is from Yeung and Yuen [21], in which the authors have constructed analytic solutions in the case with density-dependent viscosities. Similar results were further studied in [22] with or without a density jump across the boundary. Such solutions indicate that the domain of the gas (fluid) will expand as time grows up, and the density will decrease to zero everywhere including the centre.