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An analysis of an optimal control problem for mosquito populations

Cícero Alfredo da Silva Filho[a,*], José Luiz Boldrini[b]

[a] *Universidade Estadual de Santa Cruz, Ilhéus, BA, Brazil*
[b] *Universidade Estadual de Campinas, Campinas, SP, Brazil*

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ABSTRACT

We analyze a system of nonlinear partial differential equations modeling the dynamics of certain mosquito populations by taking in consideration the iterations among the immature (aquatic) subpopulation, the adult winged subpopulation and the environment resources; the immature subpopulation is assumed to be age-structured and the model also considers the action of control mechanisms on these subpopulations.

After a first analysis on the existence and uniqueness of solutions for the model, we use the obtained results to prove the existence of an optimal solution of a given optimal control problem. The corresponding first order optimality conditions are also rigorously obtained by approximating our original optimal control problem by pure optimization problems obtained by using penalization arguments.
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1. Introduction

In this article we analyze an optimal control problem associated to the following system of partial differential equations related to the one studied in Calsina e Elidrissi [1]:

$$\begin{cases} u_t(a,t) + u_a(a,t) + m_1(r(t))u(a,t) + \mu_1(c(a,t))u(a,t) = 0, \\ v'(t) + m_2(r(t))v(t) + \mu_2(L_1(c))v(t) = u(l,t), \\ r'(t) - [g(r(t)) - h(L_2(u,v))]r(t) = 0, \\ u(0,t) = b(t)v(t), \\ u(a,0) = u_0(a), \\ v(0) = v_0, \\ r(0) = r_0. \end{cases} \tag{1}$$

This system models the dynamics of a mosquito populations by taking in consideration the interaction among the immature (aquatic) form, the adult (winged) form of the mosquitos and the amount of available resources for survival.

* Corresponding author.
E-mail addresses: cicero@uesc.br (C.A. da Silva Filho), josephbold@gmail.com (J.L. Boldrini).

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The first equation is of Gurtin–MacCamy type and governs the age-structured dynamics of the immature (aquatic) mosquito population, $u = u(a,t)$, where a represents age, and t, time; here, $0 \leq a \leq l$, where $l > 0$ is given and denotes the maturation age, that is, the age when an aquatic individual becomes adult; $T > 0$ is given and denotes the maximum time of interest; we denote $Q = (0,l) \times (0,T)$. We remark that, for simplicity of the model, eggs, pupae and larval forms were lumped together in a unique aquatic population.

The second equation governs the dynamics of the adult (winged) mosquito population, $v = v(t)$, which in the present model, as in Calsina e Elidrissi [1], is considered non age-structured.

The third equation governs the variation of the amount of available resources denoted by $r(t)$.

The fourth equation in (1) is the standard renewal condition that requires that new immature individuals enter in the system due to reproduce mechanism of the adults; in this equation, $b(\cdot) \in L^\infty(0,T)$ is a positive function related to the adult fertility rate.

Moreover, as in Calsina e Elidrissi [1], in the first equation of (1), $m_1(r(t))$ is the natural mortality rate of aquatic individuals, while $m_2(r(t))$ in the second equation is the corresponding natural mortality rate of the adults; both such mortalities may depend on the available amount of resources. In the third equation, $g(\cdot)$ is a Verhurst type function associate to the possibility of recovery of the available resources, while the degradation rate of the resources by the populations is given by $h(\cdot)$, which is a known nonnegative function; such degradation is mediated by a linear integral operator L_2 to be described in the next section.

However, differently from the system in Calsina e Elidrissi [1], here we consider the action of an external control variable $c = c(a,t)$, $(a,t) \in Q$, associated for instance with the use of chemical agents, that act by increasing the mortality rates of the populations; in the case of immature individuals, such action may depend on their respective maturity level. Thus, in the first equation of (1), $\mu_1(c(a,t))$ is an additional mortality rate of the immature form caused by the external control c, while in the second equation $\mu_2(L_1(c))(t)$ is the respective additional mortality rate for the adult form; the control action in this case is mediated by a linear integral operator L_1 to be detailed described in the next section. In other words, (1) is a controlled version of the model considered by [1].

We also observe that we set the controls in such way that, by choosing suitably the given data, we may consider several situations. In particular we also have the extreme situations: by taking $\mu_1(\cdot) \neq 0$ and $\mu_2(\cdot) \equiv 0$, we are left with a problem where just the aquatic immature subpopulation is being controlled; on the contrary, by taking $\mu_1(\cdot) \equiv 0$, $\mu_2(\cdot) \neq 0$ and calling $\tilde{c} = L_1(c)(t)$ as the actual control, we are left with a problem where just the adult winged subpopulation is being controlled; in other situations both subpopulations are affected by the control.

To finish the description of the system data, we say that as expected from their meanings, the initial conditions are as follows: u_0 is a nonnegative function in $L^\infty(0,l)$; v_0 and r_0 are nonnegative numbers.

In the first part of this work, we will be concerned with the question about existence, uniqueness and estimates of such solutions. In this aspect, we recall that Calsina e Elidrissi [1], by using semigroup theory, presented a result on global existence and uniqueness of solutions for the system without the control terms and somewhat different requirements (for instance, they assume that the death rates of young and adults are decreasing functions of the amount of available resources); see also related results in Calsina and Elidrissi [2] and in Calsina and Saldaña [3]. Moreover, these authors were mainly interested in the asymptotic behavior of the solutions.

On the other hand, here we are interested in obtaining existence and uniqueness results suitable for using in optimal control problems with (1) as the state equations. With this purpose, we present a result of this type for system (1), under rather general conditions and in a sense to be precisely described in the next section. Such result is presented not only because the system analyzed in this work is a controlled one, and our conditions are somewhat different than the ones of Calsina and Elidrissi [1], but mainly because it is prepared to be used in optimal control problems with (1) as the state equations, and thus a very important aspect to be analyzed are the explicit dependence of certain estimates for such solutions on the parameters

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