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Tridispersive thermal convection

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ABSTRACT

We derive the linear instability and nonlinear stability thresholds for a problem of thermal convection in a tridispersive porous medium with a single temperature. Importantly we demonstrate that the nonlinear stability threshold is the same as the linear instability one. The significance of this is that the linear theory is capturing completely the physics of the onset of thermal convection. This result is different to the general theory of thermal convection in a tridispersive porous material where the temperatures in the macropores, mesopores and micropores are allowed to be different. In that case the coincidence of the nonlinear stability and linear instability boundaries has not been proved.

1. Introduction

A tridispersive porous medium is one where the solid skeleton contains three types of pores. One type is the usual pores which are referred to as macropores. In addition there are pores on a smaller scale referred to as mesopores, and cracks or fissures on a yet smaller scale which are referred to as micropores. The basic theory for thermal convection in a triple porosity (tridispersive) medium was developed by Nield & Kuznetsov [1]. These writers allowed for distinct velocity, temperature and pressure fields in each of the pore systems, macro, meso and micro.

The porosity associated with the macropores is denoted by ϕ, i.e. ϕ is the ratio of the volume of the macropores to the total volume of the saturated porous material. Furthermore, the mesopores generate a porosity ϵ which is the ratio of the volume occupied by the mesopores to the volume of the porous body which remains once the macropores are removed. This means that the fraction of volume occupied by the mesopores is $\epsilon(1-\phi)$. Then, the micropores generate a porosity η which is the ratio of the volume occupied by the micropores to the volume of the porous body which remains once the macro and mesopores are

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removed. This yields the fraction of volume occupied by the micropores being $\eta(1-\epsilon)(1-\phi)$ while the fraction of volume occupied by the solid skeleton is $(1-\eta)(1-\epsilon)(1-\phi)$.

Theoretical work on thermal flow in tridispersive porous media commenced with work of Nield and Kuznetsov [1]. Further work on various problems is due to Nield & Kuznetsov [2], Cheng [3], Ghalambaz et al. [4], and Straughan [5], chapter 13. Fundamental work on the thermal convection problem was developed by Kuznetsov & Nield [6]. These articles utilize different velocities U_i^f, U_i^p and U_i^c in the macro, meso and micropores, with different temperatures T^f, T^p and T^c.

Undoubtedly the current interest in tridispersive porous media is due to the many applications arising in engineering and in real life. For example, underground oil reservoirs are modelled as triple porosity systems, e.g. Ali et al. [7], Deng et al. [8], Olusola et al. [9], and Wang et al. [10]. Triple porosity features in modelling methane recovery from coal beds, Wei & Zhang [11], Zou et al. [12], and likewise is important in analysing the procurement of drinking water from an aquifer, Zuber & Motyka [13]. In a context important for the present work, triple porosity is proving very important in geothermal reservoir modelling, Siratovich et al. [14].

In this paper we develop and fully analyse thermal convection in a tridisperse porous medium when only one temperature is employed and the horizontal layer containing the saturated porous medium is heated from below. For many practical situations we believe this is sufficient. To achieve our aim it is first necessary to derive a suitable mathematical model.

2. Basic model

We begin with fields in the solid, fluid in the macropores, fluid in the mesopores, and fluid in the micropores, and we denote each phase by s, f, m and c, respectively. As stated earlier the macro, meso and micro porosities are ϕ, ϵ and η. The actual fluid velocities in the macro, meso and micropores are denoted by V_i^f, V_i^m and V_i^c and these are connected to the pore averaged velocities in the macro, meso and micro phases, U_i^f, U_i^m and U_i^c, by the relations

$$U_i^f=\phi V_i^f, \qquad U_i^m=\epsilon(1-\phi)V_i^m, \qquad U_i^c=\eta(1-\phi)(1-\epsilon)V_i^c. \tag{1}$$

Let ϵ_1, ϵ_2 and ϵ_3 be defined by

$$\epsilon_1=(1-\phi)(1-\epsilon)(1-\eta), \qquad \epsilon_2=\epsilon(1-\phi), \qquad \epsilon_3=\eta(1-\phi)(1-\epsilon). \tag{2}$$

Let the temperatures in the solid, macro, meso and micro phases be denoted by T^s, T^f, T^m and T^c, with $(\rho c)_\alpha$, κ_α, $\alpha=s,f,m$ or c, being the product of the density and specific heat at constant pressure, and the thermal conductivity, in each phase. We wish to write equations for energy balance in each phase in the tridispersive porous medium and to do this we are guided by Kuznetsov & Nield [6], equations (11)–(13) and (18)–(23), and also by the equations for a single porosity porous medium under conditions of local thermal non-equilibrium, cf. Straughan [5], equations (2.9)–(2.12).

The equations of balance of energy in the solid, macropore, mesopore, and micropore phases are then

$$\epsilon_1(\rho c)_s T^s_{,t}=\epsilon_1\kappa_s\Delta T^s+s_1(T^f-T^s)+s_2(T^m-T^s)+s_3(T^c-T^s), \tag{3}$$

$$\phi\big[(\rho c)_f T^f_{,t}+(\rho c)_f V_i^f T^f_{,i}\big]=\phi\kappa_f\Delta T^f+h_{12}(T^m-T^f)+s_1(T^s-T^f), \tag{4}$$

$$\begin{aligned}\epsilon_2\big[(\rho c)_m T^m_{,t}+(\rho c)_m V_i^m T^m_{,i}\big]=&\ \epsilon_2\kappa_m\Delta T^m+h_{12}(T^f-T^m)\\&+h_{23}(T^c-T^m)+s_2(T^s-T^m),\end{aligned} \tag{5}$$

and

$$\epsilon_3\big[(\rho c)_c T^c_{,t}+(\rho c)_c V_i^c T^c_{,i}\big]=\epsilon_3\kappa_c\Delta T^c+h_{23}(T^m-T^c)+s_3(T^s-T^c). \tag{6}$$