Nonlinear Analysis: Real World Applications 42 (2018) 387–408

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Multiplicity and concentration of positive solutions for fractional p-Laplacian problem involving concave–convex nonlinearity[☆]

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ARTICLE INFO

Article history:
Received 26 October 2017
Received in revised form 9 December 2017
Accepted 16 January 2018

Keywords:
Positive solution
Fractional p-Laplacian equation
Concentration

ABSTRACT

In this paper, we discuss a fractional p-Laplacian problem. Under suitable assumptions on nonlinearity and weight, the existence and multiplicity of positive solutions are obtained via variational method. Moreover, the global maximum point of the solution concentrates at a local minimum point of the weight function.

1. Introduction

We study the following fractional p-Laplacian problem

$$\begin{cases} \varepsilon^{ps}(-\Delta)_p^s u + V(x)|u|^{p-2}u = \lambda f(x)|u|^{q-2}u + g(x)|u|^{r-2}u \text{ in } \mathbb{R}^N, \\ u \in W^{s,p}\left(\mathbb{R}^N\right), \end{cases} \tag{F_ε}$$

where $\varepsilon, \lambda > 0$ are parameters, $N > ps$ with $s \in (0,1)$ fixed, $1 < q < p < r < p_s^*$, $p_s^* = Np/(N-ps)$. Denote by $(-\Delta)_p^s$ the fractional p-Laplacian operator, which is defined as

$$(-\Delta)_p^s u(x) = c\ P.V. \int_{\mathbb{R}^N} \frac{|u(y)-u(x)|^{p-2}(u(y)-u(x))}{|x-y|^{N+ps}} dy,$$

where c is a normalization constant depending only on N, s, p and $P.V.$ is the Cauchy principal value. Assume that the weight functions f, g and V satisfy the following conditions:

☆ This work was supported by NNSF of China (No. 11401477) and Liaoning Support Plan to the Excellent Talents (No. LJQ2014128).

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https://doi.org/10.1016/j.nonrwa.2018.01.013
1468-1218/

(F) $f \geq 0, \not\equiv 0,\ f \in L^{q^*}\left(\mathbb{R}^N\right) \cap C\left(\mathbb{R}^N\right)$ $(q^* = r/(r-q))$ with $|f|_{q^*} > 0$ and

$$f_{\max} := \max_{x \in \mathbb{R}^N} f(x) = 1.$$

(G_1) g is a positive continuous function defined on $\mathbb{R}^N$.

(G_2) There exist k points $a^1, a^2, \ldots, a^k$ in $\mathbb{R}^N$ such that

$$g(a^i) = \max_{x \in \mathbb{R}^N} g(x) = 1 \text{ for } 1 \leq i \leq k,$$

and $0 < g_\infty := \lim_{|x| \to +\infty} g(x) < 1$.

(V_1) $V \in C\left(\mathbb{R}^N, \mathbb{R}\right)$ and satisfies the following condition

$$V_\infty := \liminf_{|x| \to +\infty} V(x) > V_0 := \inf_{x \in \mathbb{R}^N} V(x) > 0.$$

(V_2) $V\left(a^i\right) = V_0,\ i = 1, 2, \ldots, k.$

In recent years, a great number of papers have been focused on studying the problems involving fractional Sobolev spaces and corresponding nonlocal equations. They naturally arise in many different contexts, such as, in phase transitions, optimization, anomalous diffusion, soft thin films, flame propagation, ultra-relativistic limits of quantum mechanics, multiple scattering, materials science and water waves, turbulence models, molecular dynamics, anomalous diffusion, minimal surfaces, quasi-geostrophic flows, conservation laws, semipermeable membranes, crystal dislocation, stratified materials, finance, the thin obstacle problem, see e.g. [1–8] and the references therein. For the basic properties of fractional Sobolev spaces, refer to [9]. As to the fractional p-Laplacian, consider the following quasi-linear problem

$$\begin{cases} (-\Delta)_p^s u = f(x, u) \text{ in } \mathbb{R}^N, \\ u \in W^{s,p}\left(\mathbb{R}^N\right). \end{cases}$$

For this kind of problem, some results have been obtained, e.g. [10–13] and the references therein. From the view of regularity theory, some results can be found in [14,15] even though that work is mostly focused on the case when p is large and the solutions inherit some regularity directly from the functional embedding themselves. Moreover, Chen and Squassina [16] studied the existence and multiplicity of non-negative solutions to the problem above when the nonlinearity is critical growth with concave–convex nonlinearities.

As for the fractional p-Laplacian operator $(-\Delta)_p^s$, when $p = 2$, it becomes the fractional Laplacian $(-\Delta)^s$. The problems with $(-\Delta)^s$ have been studied by many researchers. For example, [17] for the subcritical case and [7,18] for the critical case. In particular, Brändle, Colorado, de Pablo and Sánchez [19] studied the fractional Laplace equation involving concave–convex nonlinearity for the subcritical case. Wei and Su [20] obtained the existence and multiplicity of nontrivial solutions for the general nonlinearity by the Mountain Pass Theorem and some other nonlinear analysis methods. Moreover, by Nehari manifold and Fibering maps, the authors [21] obtained the existence and multiplicity of solutions to fractional Laplacian problem for subcritical case and critical case. In addition, $(-\Delta)_p^s$ reduces to the standard p-Laplacian $(-\Delta)_p$ as $s \to 1$, see [9]. The p-Laplacian equations have been also focused by a great number of papers, see e.g. [22–26] and the references therein.

For the problem (F_ε) with $s = 1, p = 2$ and $\lambda f(x)|t|^{q-2}t + g(x)|t|^{r-2}t$ replaced by $f(t)$, it reduces to the well-known Schrödinger equation

$$-\varepsilon^2 \Delta u + V(x)u = f(u) \text{ in } \mathbb{R}^N. \tag{1.1}$$

Recently, the existence and concentration behavior of the positive solutions of (1.1) have been researched by many authors, such as [27,28] and the references therein. In [27], the authors assumed that the potential V is continuous, positive and bounded away from zero at infinity, and there exist an open bounded set O in $\mathbb{R}^N$ and $m > 0$ such that

$$m := \inf_{x \in O} V(x) < \min_{x \in \partial O} V(x).$$