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Nonlinear evolutionary systems driven by mixed variational inequalities and its applications☆

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ABSTRACT

In this paper we investigate the system obtained by mixing a nonlinear evolutionary equation and a mixed variational inequality ((EEVI), for short) on Banach spaces in the case where the set of constraints is not necessarily compact and the problem is driven by a ϕ-pseudomonotone operator which is not necessarily monotone. In this way, we extend the recent results in Liu–Zeng–Motranu, (2016). First, it is shown that the solution set for the mixed variational inequality associated to problem (EEVI) is nonempty, closed, convex and bounded. Upper semicontinuity and measurability properties are also established. Then, relying on these results, we prove the existence of solutions for problem (EEVI) as well as a compactness property for the solution set. Finally, as an application, we study a new class of partial differential complementarity problems.

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1. Introduction

Let E, E_1 be two real Banach spaces, and let K be a nonempty closed and convex subset of E_1. Let $A : D(A) \subset E \to E$ be the infinitesimal generator of a C_0-semigroup e^{At} in E, let $\phi : K \to (-\infty, +\infty]$ be a convex, lower semicontinuous $\not\equiv +\infty$ functional, and let $f : [0, T] \times E \times K \to E$ and $g : [0, T] \times E \times K \to E_1^*$ be fixed mappings, with some constant $T > 0$.

With these data, consider the system consisting of a nonlinear evolution equation and a mixed variational inequality ((EEVI), for short)

$$\begin{cases} \dot{x}(t) = Ax(t) + f(t, x(t), u(t)) & \text{for a.e. } t \in [0, T], \\ u(t) \in SOL(K, g(t, x(t), \cdot), \phi) & \text{for a.e. } t \in [0, T], \\ x(0) = x_0, \end{cases} \tag{1.1}$$

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where $SOL(K, g(t, x(t), \cdot), \phi)$ stands for the solution set of the variational inequality: find $u = u(t) \in K$ such that

$$\langle g(t, x(t), u), v - u\rangle + \phi(v) - \phi(u) \geq 0 \text{ for all } v \in K. \tag{1.2}$$

Notice that in (1.1) it is incorporated an evolutionary variational inequality called mixed variational inequality seeking a function $u : [0, T] \to K$ that corresponds to the trajectory $x(t)$ of (EEVI).

Problem (EEVI) has been investigated for the first time in our recent work [1]. The solution to (EEVI) is understood in the following sense, [1–5].

Definition 1.1. A pair of functions (x, u), with $x \in C(0, T; E)$ and $u : [0, T] \to K$ measurable, is said to be a mild solution of system (EEVI) if

$$x(t) = e^{At}x_0 + \int_0^t e^{A(t-s)}f(s, x(s), u(s))\, ds \quad \text{for a.e. } t \in [0, T], \tag{1.3}$$

and $u(s) \in SOL(K, g(s, x(s), \cdot), \phi)$ for a.e. $s \in [0, T]$. If (x, u) is a mild solution of problem (EEVI), then $x(t)$ is called the mild trajectory and $u(t)$ is called the variational control trajectory.

Problem (1.1) originates in the study of differential variational inequalities ((DVIs), for short) as introduced by Pang–Stewart [6]. We recall that a differential variational inequality consists of an ordinary differential equation parameterized by an algebraic variable that is required to solve a finite-dimensional variational inequality containing the state variable of the system. The (DVIs) are useful for representing models that involve simultaneously dynamics and constraints in the form of inequalities arising in many applied problems such as electrical circuits with ideal diodes, Coulomb friction problems for contacting bodies, economical dynamics, dynamic traffic networks (see [7–18]). In this respect, Wang–Tang–Li-Huang [19] examined the existence of solutions for the following (DVI) in a finite-dimensional space

$$\begin{cases} \dot{x}(t) = f(t, x(t)) + B(t, x(t))u(t) \text{ for a.e. } \ t \in [0, T], \\ u(t) \in K, \text{ s.t. } (G(t, x(t)) + F(u(t)))^T \cdot (v - u(t)) \geq 0 \quad \text{for all } v \in K, \quad \text{and for a.e. } t \in [0, T], \\ x(0) = x_0, \end{cases}$$

whereas Liu–Loi–Obukhovskii [20] studied the existence of periodic solutions of the following (DVI) in $\mathbb{R}^n$

$$\begin{cases} x'(t) = \lambda f(t, x(t)) + B(t, x(t))u(t) \quad \text{for a.e. } t \in [0, T], \\ u(t) \in K, \text{ s.t. } (G(t, x(t)) + F(u(t)))^T \cdot (v - u(t)) \geq 0 \quad \text{for all } v \in K, \quad \text{and for a.e. } t \in [0, T], \\ x(0) = x(T). \end{cases}$$

Other results of this type in finite-dimensional spaces can be found in [21–26].

The first work focusing on the (DVIs) in infinite-dimensional spaces is due to Liu–Zeng–Motreanu [1], where the general inequality problem (EEVI) in (1.1) on Banach spaces is considered when K is a compact set. This represents extensions of [6,20,23]. A serious limitation of the applicability of the results in [1] is the fact that the constraint set K is required to be compact.

The purpose of this paper is to study, on the basis of [1], the properties of solution set for the general problem (EEVI) with (1.1) in Banach spaces in the case where K is nonempty, closed and convex, not necessarily compact. In this noncompact setting, we are able to prove a general existence theorem for mixed variational inequalities, strong–weak upper semicontinuity and measurability properties for their solution sets, existence of solutions and compactness of the solution set for (EEVI). Finally, through the abstract results for (EEVI), we treat a new topic termed as partial differential complementarity problem. We emphasize that the results obtained here cannot be derived from the work in [1] due to the noncompactness

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