Nonlinear Analysis: Real World Applications 42 (2018) 422–447

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

# Mathematical modeling of biofilm development

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ARTICLE INFO

*Article history:*
Received 20 March 2017
Received in revised form 7 December 2017
Accepted 2 January 2018

*Keywords:*
Navier–Stokes
Degenerate diffusion
Obstacle
Flow
Variational inequality
Subdifferential

ABSTRACT

We perform mathematical analysis of the biofilm development process. A model describing biomass growth is proposed: It arises from coupling three parabolic nonlinear equations: a biomass equation with degenerate and singular diffusion, a nutrient transport equation with a biomass-density dependent diffusion, and an equation of the Navier–Stokes type, describing the fluid flow in which the biofilm develops. This flow is subject to a biomass' density-dependent obstacle. The model is treated as a system of three inclusions, or variational inequalities; the equation of the Navier–Stokes type causes major difficulties for the system's solvability. Our approach is based on the recent development of the theory on Navier–Stokes variational inequalities.

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## 1. Introduction

It is quite important for our future to find clean and reproducible materials and energy resources. In this connection, biomass has been noticed for the last thirty years. Biomass growth is a process of aggregation of some living organisms transported in fluids (liquids or gases), usually sticking to the walls of the fluid container, and thus influencing the flow itself. It also involves nutrient transport and consumption. It can occur in air, water, soil penetrated by any fluid, blood. Only little is known about mathematical models of this mechanism. In particular, the process occurs in fluids, but models coupled with hydrodynamics have been seldom analyzed.

In [1], such a biomass growth model coupled with fluid dynamics has been proposed in the three dimensional space. However, as far as we know, no theoretical results appeared in this context. The model assumed a sharp interface between the (solid) biomass and the liquid. In the present paper we propose an analogous mathematical model of biomass growth dynamics in a fluid, postulating, in place of a sharp interface, a thicker layer, considered as a mixture of both phases — just as in the weak formulation of a solid–liquid phase transition.

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https://doi.org/10.1016/j.nonrwa.2018.01.005
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For other formulations of biomass growth with taxis terms, see [2]. These formulations are not explicitly included in our formulation, but can be easily obtained by a modification.

Let us recall in more detail the mathematical full model proposed in [1]. Let $\Omega \subset \mathbf{R}^3$ be a container in which biomass growth takes place. The process is described in terms of three unknown functions $\boldsymbol{v}(x,t)$, $w(x,t)$ and $u(x,t)$ which are respectively the velocity of the fluid, the nutrient concentration and the biomass density at a point $x \in \Omega$ and time $t \geq 0$. They are governed by the following system:

$$
\begin{aligned}
&(H_0)\ \boldsymbol{v}_t + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \nu\Delta\boldsymbol{v} = -\frac{1}{\rho}\nabla P,\ \text{div}\ \boldsymbol{v} = 0,\ \text{in}\ \{(x,t)|u(x,t)=0\},\\
&\qquad\qquad \text{where } \rho \text{ is the constant density and } P \text{ is the pressure in the fluid,}\\
&(N_0)\ w_t + \boldsymbol{v}\cdot\nabla w - \text{div}\ (d(u)\nabla w) = -f(w)u \text{ in } \Omega,\ t>0,\\
&\qquad\qquad \text{where } f(w) = \frac{k_1 w}{k_2 + w} \text{ for positive constants } k_1,\ k_2,\\
&(B_0)\ u_t - \Delta d_1(u) + bu = f(w)u \ \text{ in } \Omega,\ t>0,
\end{aligned}
$$

subject to suitable initial and boundary conditions. This model is derived under the postulate that the fluid cannot penetrate into the solid biomass ($u > 0$), the nutrient is convective by $\boldsymbol{v}\cdot\nabla w$ and diffusive with biomass-density dependent coefficient $d(u)$, and the diffusion of biomass is very slow near the interface $u = 0$, but very fast near the maximum density $u = u^*$. The function $f$ is the nutrient consumption term and $b$ is a positive constant.

In this paper, we propose some relaxations and modifications into the above model, postulating that:

(i) The biomass density $u(x,t)$ is non-negative and it has the finite maximum value $u^*$, i.e. $0 \leq u(x,t) \leq u^*$. For some $\delta_0 \in (0,u^*)$, which is fixed, we postulate that the region of high density $\delta_0 \leq u(x,t) \leq u^*$ is solid, and that of low density $0 < u(x,t) < \delta_0$ is the interface layer between the solid biomass and the liquid. In such a layer, the behavior of $u$ may correspond to the dynamics of planktonic biomass floating in the liquid, cf. e.g. [3].
This causes a biomass dependent constraint on the fluid's velocity. The constraint is written as:

$$|\boldsymbol{v}(x,t)| \leq p_0(u^\varepsilon(x,t)),$$

where $p_0(r) : (0,u^*] \to [0,\infty)$ is a $C^1$, non-negative and non-increasing function on $(0,u^*]$ such that (see Fig. 1(i)):

$$\lim_{r\downarrow 0} p_0(r) = \infty,\quad p_0'(r) < 0,\ \forall r \in (0,\delta_0),\quad p_0(r) = 0,\ \forall r \in [\delta_0, u^*]; \tag{1.1}$$

on the other hand $u^\varepsilon := \rho_\varepsilon * u$ is the local spatial-average of $u(x,t)$ by means of the usual mollifier $\rho_\varepsilon(x)$ (see Section 2 for details).

(ii) The nutrient concentration $w(x,t)$ is non-negative and has the threshold value 1, i.e. $0 \leq w(x,t) \leq 1$. Also, we suppose that there is no nutrient supply from the exterior. The diffusion coefficient $d(u)$ depends on the biomass density $u$ and

$$c_d \leq d(r) \leq c_d',\quad |d(r_1) - d(r_2)| \leq L(d)|r_1 - r_2|,\quad \forall r_1,\ r_2 \in \mathbf{R}, \tag{1.2}$$

where $c_d,\ c_d'$ and $L(d)$ are positive constants (see Fig. 1(ii)). The function $f(w)u$, appearing in biomass density and nutrient transport equations, is called the nutrient consumption, and in our model we suppose that

$$f(w)\ \text{ is of } C^1 \text{ and Lipschitz in } w \in \mathbf{R},\ f(0) \leq 0 \text{ and } f(1) \geq 0. \tag{1.3}$$

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