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Homoclinic orbits for a class of first order nonperiodic Hamiltonian systems

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ABSTRACT

In this paper we study the existence of infinitely many homoclinic orbits for a class of first order Hamiltonian systems $\dot{z} = JH_z(t,z)$, where the Hamiltonian function $H(t,z)$ is nonperiodic in t and only locally defined near the origin with respect to z. Under some mild conditions on H, we show that these Hamiltonian systems possess a sequence of homoclinic orbits near the origin.

1. Introduction and main results

Let us consider the first order Hamiltonian system

$$\dot{z} = \mathcal{J}H_z(t,z), \tag{HS}$$

where $z = (p,q) \in \mathbb{R}^N \times \mathbb{R}^N = \mathbb{R}^{2N}$ and

$$\mathcal{J} = \begin{pmatrix} 0 & -I_N \\ I_N & 0 \end{pmatrix}$$

is the standard symplectic matrix with I_N being the identity matrix in $\mathbb{R}^N$. Here the Hamiltonian function $H(t,z)$ has the form

$$H(t,z) = \frac{1}{2}L(t)z \cdot z + G(t,z) \tag{1.1}$$

with L being a continuous symmetric $2N \times 2N$ matrix-valued function. As usual, we say that a solution z of (HS) is a homoclinic orbit if $z(t) \not\equiv 0$ and $z(t) \to 0$ as $|t| \to \infty$.

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By virtue of variational methods, the existence and multiplicity of homoclinic orbits for (HS) have been extensively investigated in the literature over the past several decades. Most of the results are obtained under the assumptions that the Hamiltonian function $H(t,z)$ depends periodically on t and $G(t,z)$ satisfies various growth conditions at infinity with respect to z, see, for instance, [1–14] and the references therein.

Without the assumption of periodicity, the problem is quite different in nature due to the lack of compactness of Sobolev embedding. In the early paper [15], the authors first studied the existence of homoclinic orbits for (HS) in the nonperiodic case provided that L has a special form and $G(t,z)$ satisfies some kind of superquadratic or subquadratic growth conditions at infinity with respect to z. Indeed, the problem recovers partial compactness under some fairly strong assumptions on L there. Subsequently, the authors in [16] also obtained the existence and multiplicity of homoclinic orbits for (HS) in this case by assuming that L satisfies a more general condition than those in [15] and $G(t,z)$ is asymptotically quadratic at infinity with respect to z. After the work of [16], there are also a few papers devoted to the nonperiodic case (see, e.g., [17–21]). We note that in all these papers the function $G(t,z)$ was always required to satisfy various growth conditions at infinity with respect to z, which is crucial for the corresponding results.

In the present paper, motivated by [16] and [22], we are going to study the existence of infinitely many homoclinic orbits for (HS) without any assumption on the function $G(t,z)$ at infinity with respect to z. Before describing our result, we give some notations as in [16]. For two given $2N\times 2N$ matrices L_1 and L_2, we say that $L_1 \geq L_2$ if and only if

$$\min_{\zeta\in\mathbb{R}^{2N},\,|\zeta|=1}(L_1-L_2)\zeta\cdot\zeta \geq 0$$

and that $L_1 \ngeq L_2$ if and only if $L_1 \geq L_2$ does not hold. Let

$$\mathcal{J}_0 = \begin{pmatrix} 0 & I_N \\ I_N & 0 \end{pmatrix}$$

and I_{2N} be the identity matrix in $\mathbb{R}^{2N}$. Then we present the following assumptions.

($\mathrm{H}_0^{\pm}$) There exist constants $r_0>0$ and $b_0>0$ such that

$$\lim_{|s|\to\infty}\operatorname{meas}\{t\in(s-r_0,s+r_0)\mid \pm\mathcal{J}_0L(t)\ngeq b_0I_{2N}\}=0,$$

where $\operatorname{meas}(\cdot)$ denotes the Lebesgue measure in $\mathbb{R}$.

(H_1) $G\in C^1(\mathbb{R}\times B_\delta(0),\mathbb{R})$ is even in z and $G(t,0)\equiv 0$, where $B_\delta(0)$ is the ball in $\mathbb{R}^{2N}$ centered at 0 with radius $\delta>0$.

(H_2) There exist constants $\nu\in(0,1)$, $\mu\in(2,2/(1-\nu)]$ and a nonnegative function $\xi\in L^\mu(\mathbb{R},\mathbb{R})$ such that

$$|G_z(t,z)|\leq \xi(t)|z|^\nu \quad \text{for all } (t,z)\in\mathbb{R}\times B_\delta(0).$$

($\mathrm{H}_3^{\pm}$) There exist a constant $c_1>0$ and a nonnegative function $\eta\in L^\mu(\mathbb{R},\mathbb{R})$ such that

$$0<\eta(t)\leq c_1\xi(t) \quad \text{for a.a. } t\in\mathbb{R}$$

and

$$\lim_{|z|\to 0}\frac{G(t,z)}{\eta(t)|z|^2}=\pm\infty \text{ uniformly for a.a. } t\in\mathbb{R}.$$

Our main result reads as follows.

Theorem 1.1. *Suppose that* (H_0^+) (*or* (H_0^-)), (H_1), (H_2) *and* (H_3^+) (*or* (H_3^-)) *are satisfied. Then* (HS) *possesses a sequence of homoclinic orbits* $\{z_k\}$ *such that* $\max_{t\in\mathbb{R}}|z_k(t)|\to 0$ *as* $k\to\infty$.