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Viscous–capillary traveling waves associated with classical and nonclassical shocks in van der Waals fluids

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ABSTRACT

We present in this work existence results for various types of diffusive–dispersive traveling waves associated with shock waves with the same shock speed in van der Waals fluids. For any shock satisfies a single entropy inequality, there exist corresponding traveling waves with suitable choices of viscosity and capillarity coefficients. The shock may be a classical shock, a nonclassical shock satisfying Lax's shock inequalities, or a nonclassical shock violating Lax's shock inequalities. Therefore, shock waves satisfying a single entropy inequality are also admissible under the criterion of vanishing viscosity/capillarity effects.

1. Introduction

Shock waves in van der Waals fluids have many applications. Shocks are discontinuous solutions and it is interesting to study their approximations by smooth solutions of the system obtained by allowing viscosity and capillarity effects. Such typical smooth approximate solutions are known as *traveling waves*.

We aim in this paper to show that any shock satisfying a given entropy inequality can be approximated by diffusive–dispersive traveling waves. A typical model is the one of fluid dynamics equations in the Lagrangian coordinates for an isentropic van der Waals fluid with nonlinear diffusion and dispersion coefficients

$$\begin{aligned} &\partial_t v - \partial_x u = 0, \\ &\partial_t u + \partial_x p(v) = \varepsilon(a(v)|v_x|^q u_x)_x - \delta v_{xxx}, \end{aligned} \tag{1.1}$$

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where $u > 0, v > 0$ and p denote the velocity, specific volume and the pressure, respectively; $a = a(v)$ is a viscous function, q is a nonnegative real number, and $\varepsilon > 0, \delta > 0$ are positive viscosity and capillarity coefficients, see [1]. In the sequel we will assume that ε and δ are constant for simplicity. It has been known that whenever traveling waves of (1.1) exist, their limit when $(\varepsilon, \delta) \to (0,0)$ is a shock wave of the following system of conservation laws

$$\begin{aligned} &\partial_t v - \partial_x u = 0, \\ &\partial_t u + \partial_x p(v) = 0, \end{aligned} \tag{1.2}$$

which satisfies the entropy condition

$$\begin{aligned} &\partial_t U(u,v) + \partial_x F(u,v) \le 0, \\ &U(u,v) := \frac{u^2}{2} + \Sigma(v), \quad F(u,v) := u\,p(v), \Sigma(v) := -\int_0^v p(w)\,dw, \end{aligned} \tag{1.3}$$

(in the sense of distribution). Conversely, if a shock wave of (1.2) of the form

$$(v,u)(x,t) = \begin{cases} (v_-, u_-), & x < st, \\ (v_+, u_+), & x > st, \end{cases} \tag{1.4}$$

satisfies the entropy condition (1.3), it is worth to study the existence of traveling waves of (1.1) connecting $(v_\pm, u_\pm)$. The most interesting scenario takes place when the graph of the pressure of a van der Waals fluid, as a function of the specific volume, meets the straight line through $(v_\pm, p(v_\pm))$ at four distinct points. Then, a shock wave from a given left-hand state (v_-, u_-) and a given shock speed s may define three right-hand states (v_+, u_+). Precisely, the v-value of the right-hand state can be any of the three v-component of the three intersection points $(v_i, p(v_i)), i = 1,2,3$, of the secant line passing through the point $(v_-, p(v_-))$ and the graph of the pressure function $p = p(v)$. The u-value $u = u_i, i = 1,2,3$, of the shock can be determined by the Rankine–Hugoniot relations. These three shock waves can be classified into three categories:

(a) a classical shock (see [2,3]) from the left-hand state (v_-, u_-) to the right-hand state $(v_+, u_+) = (v_1, u_1)$;
(b) a nonclassical shock (see [4–7]) violating Lax's shock inequalities from the left-hand state (v_-, u_-) to the right-hand state $(v_+, u_+) = (v_2, u_2)$;
(c) a nonclassical shock satisfying Lax's shock inequalities from the left-hand state (v_-, u_-) to the right-hand state $(v_+, u_+) = (v_3, u_3)$.

Our goal is to show that there always exist traveling waves for the shock in (a), and if the shock in (b) is admissible under the strict entropy inequality (1.3), then the shock in (c) is also admissible and traveling waves exist for both (b) and (c) .

Throughout, we assume that the viscous function a is constant, and the pressure function $p = p(v)$ is a twice differentiable and has a van der Waals type in the sense that there exist two numbers $0 < a_1 < a_2$ such that

$$\begin{aligned} &p'(v) < 0, \quad v > 0, \\ &p''(v) > 0, \quad v \in (0, a_1) \cup (a_2, +\infty), \\ &p''(v) < 0, \quad v \in (a_1, a_2), \\ &\lim_{v\to 0} p(v) = +\infty, \qquad \lim_{v\to+\infty} p(v) \ge 0, \end{aligned} \tag{1.5}$$

where $(.)' = \frac{d(.)}{dv}, (.)'' = \frac{d^2(.)}{dv^2}$.

Shock waves and traveling waves have attracted many authors. Traveling waves for diffusive–dispersive scalar equations were studied in [8,9]. A pioneering work on the related shock layers of the gas dynamics equations with viscosity and heat conduction effects (with zero capillarity) was presented in [10].