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Existence and non-existence of traveling wave solutions for a nonlocal dispersal SIR epidemic model with nonlinear incidence rate

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ABSTRACT

A nonlocal dispersal SIR epidemic model with nonlinear incidence rate is introduced. It is shown that the existence and non-existence of nontrivial and nonnegative traveling wave solutions of this model are fully determined by the threshold values, that is, the basic reproduction number $\mathcal{R}_0$ and the minimal wave speed c^*. For $\mathcal{R}_0 > 1$ and $c \geq c^*$, the existence theorem is obtained by the method of auxiliary system, Schauder's fixed point theorem and three limiting arguments. For $\mathcal{R}_0 > 1$ and $0 < c < c^*$, the non-existence theorem is derived by applying the two-sided Laplace transform and making full use of the structure of the model. For $\mathcal{R}_0 = 1$ with $c > 0$ and $\mathcal{R}_0 < 1$ with $c > 0$, the non-existence theorems are also established.

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1. Introduction

It is known that the reaction–diffusion systems have been applied to describe a variety of phenomena in epidemiology and ecology [1–10]. However, nonlocal dispersal is better described as a long range process rather than as a local one in many situations such as in population ecology, materials science, phase transition, genetics, neurology and epidemiology [11–18], and nonlocal dispersal models have attracted much attention [19–40]. Note that in modeling of infectious disease, nonlinear incidence rates have played a vital role in ensuring that the model can give a reasonable qualitative description for the disease dynamics such as cholera epidemic spread in Bari in 1973 [41,42].

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Based on the above concerns, in this paper, we introduce a nonlocal dispersal SIR epidemic model with nonlinear incidence rate

$$\begin{cases} \dfrac{\partial S(x,t)}{\partial t} = d_1(J * S(x,t) - S(x,t)) - f(S(x,t))g(I(x,t)), \\ \dfrac{\partial I(x,t)}{\partial t} = d_2(J * I(x,t) - I(x,t)) + f(S(x,t))g(I(x,t)) \\ \qquad\qquad -\gamma I(x,t), \\ \dfrac{\partial R(x,t)}{\partial t} = d_3(J * R(x,t) - R(x,t)) + \gamma I(x,t), \end{cases} \tag{1.1}$$

where $S(x,t)$, $I(x,t)$ and $R(x,t)$ denote the densities of susceptible, infective and removed individuals at time t and location x, respectively. The parameters $d_i > 0$ $(i = 1, 2, 3)$ describe the spatial motility of each class, γ stands for the recovery rate of the infective individuals and $J * S(x,t)$, $J * I(x,t)$ and $J * R(x,t)$ represent the standard convolution with space invariable x, namely,

$$J * u(x,t) = \int_{\mathbb{R}} J(x-y)u(y,t)dy = \int_{\mathbb{R}} J(y)u(x-y,t)dy,$$

where u can be either S, I or R. Throughout this paper, we always assume the nonlinear functions f, g and the kernel J satisfy the following hypotheses:
(**H1**) $f(S)$ is positive and continuous for $S > 0$ with $f(0) = 0$ and $f'(S)$ is positive and bounded for $S \geq 0$ with $L := \max_{S\in[0,\infty)} f'(S)$;
(**H2**) $g(I)$ is positive and continuous for $I > 0$ with $g(0) = 0$, $g'(I) > 0$ and $g''(I) \leq 0$ for $I \geq 0$;
(**H3**) $J \in C^1(\mathbb{R}), J(y) = J(-y) \geq 0$, $\int_{\mathbb{R}} J(y)dy = 1$ and J is compactly supported;
(**H4**) $\lim\limits_{\lambda\to+\infty} \frac{\lambda}{\int_{\mathbb{R}} J(y)e^{-\lambda y}dy} = 0$.

Taking a first order approximation by Fourier transform and Taylor formula [43,44], model (1.1) is changed to a reaction–diffusion SIR model

$$\begin{cases} \dfrac{\partial S(x,t)}{\partial t} = d_1 \dfrac{\partial^2 S(x,t)}{\partial x^2} - f(S(x,t))g(I(x,t)), \\ \dfrac{\partial I(x,t)}{\partial t} = d_2 \dfrac{\partial^2 I(x,t)}{\partial x^2} + f(S(x,t))g(I(x,t)) - \gamma I(x,t), \\ \dfrac{\partial R(x,t)}{\partial t} = d_3 \dfrac{\partial^2 R(x,t)}{\partial x^2} + \gamma I(x,t), \end{cases} \tag{1.2}$$

where f and g satisfy (**H1**) and (**H2**), respectively. By introducing an auxiliary system and applying Schauder's fixed point theorem and a limiting argument, Bai and Wu [1] proved that the subsystem of (1.2) admits a traveling wave solution $(S(x+ct), I(x+ct))$ satisfying $S(-\infty) = S_{-\infty}$, $I(\pm\infty) = 0$ and $S(+\infty) = S_\infty < S_{-\infty}$ if $\mathcal{R}_0 = f(S_{-\infty})g'(0)/\gamma > 1$ and $c > c^*$. On the contrary, if $0 < c < c^*$ or $\mathcal{R}_0 \leq 1$, they applied the two-sided Laplace transform which was first introduced by Carr [45] to obtain the non-existence of traveling wave solutions for the subsystem of (1.2).

In (1.1), the choice of $f(S) = S$ and $g(I) = I$ leads to the model

$$\begin{cases} \dfrac{\partial S(x,t)}{\partial t} = d_1(J * S(x,t) - S(x,t)) - \beta S(x,t)I(x,t), \\ \dfrac{\partial I(x,t)}{\partial t} = d_2(J * I(x,t) - I(x,t)) + \beta S(x,t)I(x,t) - \gamma I(x,t), \\ \dfrac{\partial R(x,t)}{\partial t} = d_3(J * R(x,t) - R(x,t)) + \gamma I(x,t), \end{cases} \tag{1.3}$$

which was considered in [39]. The parameter $\beta > 0$ denotes the infection rate. Yang et al. [39] established the existence of nontrivial and nonnegative traveling wave solution by constructing an invariant cone in a

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