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Mathematical modelling for the transmission of dengue: Symmetry and travelling wave analysis

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ABSTRACT

In this paper we propose some mathematical models for the transmission of dengue using a system of reaction–diffusion equations. The mosquitoes are divided into infected, uninfected and aquatic subpopulations, while the humans, which are divided into susceptible, infected and recovered, are considered homogeneously distributed in space with a constant total population. We find Lie point symmetries of the models and we study theirs temporal dynamics, which provides us the regions of stability and instability, depending on the values of the basic offspring and the basic reproduction numbers. Also, we calculate the possible values of the wave speed for the mosquitoes invasion and dengue spread and compare them with those found in the literature.

1. Introduction

The *Aedes aegypti* mosquito is a well known vector for the transmission of diseases to humans such as *dengue* and *Zika*, to name a few. Until 2015 these mosquitoes were mostly related to the transmission of dengue. However, evidence suggests that after 2014 FIFA World Cup tournament, Zika virus arrived at South America, finding in Brazil an ideal habitat to grow: a tropical climate, significantly higher population density and an efficient vector for transmission: *Aedes aegypti* [1,2]. Zika usually causes mild symptoms in most people infected by it. In spite of everything, new data gathered since the end of 2015 from women that got infected – while they where on the last months of their pregnancy – supported the suspicion that Zika is related to microcephaly, a medical condition where the baby's brain does not develop properly.

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To the best of our knowledge no mathematical models of Zika have been proposed or validated so far [3]. On the contrary, things are quite different with dengue. For dengue, Aedes mosquito is the primary vector of transmission, and therefore, the study of its dynamics is very important as it permits the determination of the efficacy of different ways of controlling the mosquitoes populations. Furthermore, as a mosquito becomes a carrier of the virus only by biting an already infected human, the transmission can be fully understood only by taking also into account the human populations. On the other hand, for Zika, this is only one of the possible ways of transmission since it can also be transmitted through other ways [3]. Nevertheless, the study of dengue's transmission may be useful not only for its own sake, but it can also enlighten and provide insights and inspiration to the mathematical understanding of Zika too.

Our paper is concerned with the mathematical modelling for transmission of dengue. In Section 2 we propose Malthusian models taking into account a division of human population into three groups (SIR classification): susceptible, infectious and recovered, while the mosquitoes are divided into female winged non-infected and infected, and aquatic sub-populations.

To have a picture of some mathematical features of the biological constitutive parameters of the models considered we look for some point symmetries of the models in Section 3. Next, in Section 4 we consider the temporal dynamics of the models. This enables us to determine the equilibrium points of the systems and determine whether these points are stable or not. In Section 5, using the invariance under space and time translations, we determine the wave speed for the mosquitoes' invasion and dispersion. To determine these values we made use of the data used in [4]. Finally, discussions and conclusions are presented in Section 6.

2. The models

We start by introducing the models for transmission of dengue relating humans and *Aedes aegypti* mosquitoes dynamics.

The human population is divided into three sub-populations: susceptible, infected and recovered individuals at a time t and a position x. The corresponding density functions are denoted by $\bar{h}(x,t)$, $\bar{I}(x,t)$ and $\bar{r}(x,t)$, respectively. By $\bar{N}(x,t)$ we designate the total human population, that is $\bar{N}(x,t) = \bar{h}(x,t) + \bar{I}(x,t) + \bar{r}(x,t)$.

The mosquitoes' population is also divided into three: winged non-infected $\bar{u}(x,t)$ and infected $\bar{w}(x,t)$ female mosquitoes and aquatic $\bar{v}(x,t)$. The latter population includes the eggs, larvae and pupae stages of *Aedes* life cycle. The total winged mosquito population is denoted by $\bar{M}(x,t)$. We only consider females in the winged populations because dengue is transmitted to humans when an infected Aedes female bites a human being to fertilise its eggs. Additionally, in this paper we assume that the reproduction of the insects is only made by oviposition of the winged females.

The biological parameters used in our models are presented in Table 1.

2.1. Previous models

Here we recall previous models that influenced this study.

2.1.1. Aedes aegypti population models

As in the model proposed in [5], here we consider only two sub-populations: the winged form, comprised of mature female mosquitoes, and an aquatic sub-population, including eggs, larvae and pupae. The spatial density of the winged population is $\bar{M}(x,t) = \bar{u}(x,t)$ and the aquatic sub-population is $\bar{v}(x,t)$. The rate of maturation from the aquatic form to the winged one, denoted by $\bar{\gamma}$, is saturated by the carrying capacity $\bar{k}_1$, given by $\bar{\gamma}\bar{v}(x,t)\left(1 - \bar{u}(x,t)/\bar{k}_1\right)$.