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Traveling wave phenomena of n-dimensional diffusive predator–prey systems

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A B S T R A C T

In this work, we deal with the n-dimensional reaction-diffusive predator–prey systems with the discrete time delay, which have numerous applications in biology and ecology. By applying an asymptotic analysis, the Schauder's fixed point theorem as well as the upper and lower solution method, we investigate the existence, non-existence, exponentially asymptotic stability and asymptotic behaviors of traveling wave solutions with the appropriate small delay when the wave speed c varies. Moreover, we derive the minimal wave speed c^* based on the well-established properties of the Fisher equation, and prove the existence of traveling wave solutions when the wave speed $c \geq c^*$, which provides a sharp contrast to the case of $c < c^*$. The obtained results extend the existing ones in the literature to arbitrary finite dimensional systems with the discrete time delay and demonstrate asymptotic behaviors of traveling wave solutions.

1. Introduction

In ecological dynamics, in-depth understanding of behaviors of interacting species has become a central issue in recent decades. Intra-actions within species are intricate, and the most common system used to describe behaviors of interacting species is the Lotka–Volterra type model, which has become one of main research topics in population dynamics [1]. Such kind of models also has widespread applications in ecological balance, fauna and flora protection and environmental governance. There induce numerous model problems that lead to an important, special class of solutions called traveling wave solutions. Examining the behaviors of these solutions can give insights into the role of corresponding mechanisms or population dynamics of the specific biological problems, and can help us to better understand how biological and ecological models are applied in the realistic situations [2,3].

As we have seen, among all intra-actions, predator–prey relations have been widely studied in the community of mathematical biology. In particular, traveling wave phenomena of predator–prey systems

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with different response functions have been extensively discussed [4–6]. For example, Pielou [1] and Malchow et al. [7] considered predator–prey intra-actions with the logistic growth. Lin et al. [8] studied the following temporal–spatial system:

$$\begin{cases} u_{1t}(x,t) = d_1 u_{1xx}(x,t) + r_1 u_1(x,t)\left[1 - u_1(x,t) - bu_2(x,t)\right], \\ u_{2t}(x,t) = d_2 u_{2xx}(x,t) + r_2 u_2(x,t)\left[-1 - u_2(x,t) + fu_1(x,t)\right], \end{cases}$$

where the predator u_2 only feeds on the prey u_1. However, in nature, one predator may have multiple choices on various foods. Thereby, Huang and Lin [9] considered a diffusive system with two preys and one predator:

$$\begin{cases} \dfrac{\partial u_1(x,t)}{\partial t} = d_1 \Delta u_1(x,t) + u_1(x,t)\left[a_1 - u_1(x,t) - c_{12}u_2(x,t) - c_{13}u_3(x,t)\right], \\ \dfrac{\partial u_2(x,t)}{\partial t} = d_2 \Delta u_2(x,t) + u_2(x,t)\left[a_2 - c_{21}u_1(x,t) - u_2(x,t) - c_{23}u_3(x,t)\right], \\ \dfrac{\partial u_3(x,t)}{\partial t} = d_3 \Delta u_3(x,t) + u_3(x,t)\left[a_3 + c_{31}u_1(x,t) + c_{32}u_2(x,t) - u_3(x,t)\right], \end{cases} \tag{1.1}$$

in which $x \in \mathbb{R}$ and $t > 0$, and all parameters except a_3 are positive. Here, when $a_3 > 0$, u_3 is considered to be a generalist with resources other than u_1 and u_2. While $a_3 < 0$, u_3 is assumed to have no other resources besides u_1 and u_2. As a result, if the preys vanish, the predator will extinct eventually. Huang and Lin [9] established the existence, nonexistence and the minimal speed of traveling wave solutions of system (1.1) under the assumption $a_3 > 0$ by applying the Schauder's fixed point theorem as well as the method of upper and lower solutions.

In view of system (1.1) in the case of $a_3 > 0$ with its biological background, Shang et al. [10] further extended results in [9] to the case where the existence of traveling wave solutions of n-dimensional delayed reaction–diffusion systems with $n-1$ preys and one predator, was investigated. That is, system (1.1) with $a_3 > 0$ becomes

$$\begin{cases} \dfrac{\partial u_i(x,t)}{\partial t} = d_i \Delta u_i(x,t) + r_i u_i(x,t)\left[1 - u_i(x,t) - \displaystyle\sum_{j=1,\, i\neq j}^{n} a_{ij}u_j(x,t-\tau_{ij})\right], \\ \dfrac{\partial u_n(x,t)}{\partial t} = d_n \Delta u_n(x,t) + r_n u_n(x,t)\left[1 - u_n(x,t) + \displaystyle\sum_{j=1}^{n-1} a_{nj}u_j(x,t-\tau_{nj})\right], \quad x \in \mathbb{R},\ t > 0, \end{cases} \tag{1.2}$$

where $i = 1, 2, \ldots, n-1$ and $\tau_{ij} \geq 0$ $(j = 1, 2, \ldots, n)$.

Numerous attention deals with the existence of traveling wave solutions in a system where spatial diffusion and temporal delay play a crucial role in dynamics of determining systems [11–13]. It has been shown that delay may induce some differences of traveling wave solutions between the delayed and undelayed systems, for example, the minimal wave speed [14,15] and the monotonicity of traveling wave solutions in scalar equations [16,17]. Moreover, Lv and Wang [2], and Hou and Zhao [18] showed that asymptotic behaviors of traveling wave solutions, which are often formulated by the asymptotic boundary conditions, usually provide us physical or biological interpretations in different natural environments. Therefore, understanding the asymptotic behavior is a fundamental issue in the study of traveling wave phenomena.

Note that in [9] it was only analyzed in the case of $a_3 > 0$, and left an open problem that if $a_3 < 0$, the construction of upper and lower solutions would be more challenging than that of the case of $a_3 > 0$. On the other hand, in [10] the existence of traveling wave solutions was only established in the case of $c > c^*$ and asymptotic behaviors of traveling wave solutions were not presented.

Denote the index sets by

$$I = \{1, 2, \ldots, n-1\} \quad \text{and} \quad J = \{1, 2, \ldots, n\}.$$