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Limit cycle bifurcations by perturbing non-smooth Hamiltonian systems with 4 switching lines via multiple parameters☆

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In this paper, we investigate the limit cycle bifurcation in perturbations of non-smooth Hamiltonian systems with 4 switching lines via multiple parameters. Using the first order Melnikov function, we derive some computational formulas, which can be used to study the bifurcation of limit cycles. As an application to the obtained results, we consider the limit cycle bifurcation problem of a non-smooth system studied in Wang and Han (2016). Comparing with the result in the above reference, we find $n-1,\ n \in \mathbb{N}^+$ more limit cycles by our method.

1. Introduction

The limit cycle bifurcation problem is an important part of the qualitative theory of differential systems, which is closely related to the well-known Hilbert's 16th problem, see [1]. Although there are many good papers researching it, see [2–4] and references therein, this problem still turns out to be a strongly difficult one (see [5,6]). In order to make this problem more simple, many researchers have began to study some special kinds of polynomial differential systems and obtain some new and good results, such as Liénard systems [7,8], near-Hamiltonian systems [9–11] and so on.

For more detailed, consider the following differential system

$$\dot{x} = H_y(x,y) + \varepsilon p(x,y) \quad \dot{y} = -H_x(x,y) + \varepsilon q(x,y), \tag{1.1}$$

where $\varepsilon > 0$ is a small parameter, $H(x,y)$, $p(x,y)$ and $q(x,y)$ are polynomials in the variables x and y. System (1.1) is called a near-Hamiltonian system and system $(1.1)|_{\varepsilon=0}$ is a Hamiltonian system. Suppose that the unperturbed system $(1.1)|_{\varepsilon=0}$ has a family of periodic orbits denoted by $\Gamma_h,\ h \in \mathcal{I}$. Then, by

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Chapter 3.1 of [2], associated to the periodic orbits $\Gamma_h,\ h \in \mathcal{I}$, we have the first order Melnikov function of system (1.1)

$$M(h) = \oint_{\Gamma_h} q(x,y)dx - p(x,y)dy, \quad h \in \mathcal{I}, \tag{1.2}$$

which, by Chapter 2.1 in [10], is also called an Abelian integral. As a matter of fact, the function in (1.2) is a coefficient of the bifurcation function of system (1.1) up to first order of ε, which can be used to discuss the local and global bifurcation problems.

Recently, stimulated by the non-smoothness appearing in many real models, there has been considerable interest in study of non-smooth differential systems, see [12] and references therein. As we have been seen, many methods for the smooth ones can be generalized to discuss the non-smooth ones, see [13–15] and references cited here. For definiteness, consider a non-smooth differential system of the form

$$\begin{cases} \dot{x} = H_y^-(x,y) + \varepsilon p^-(x,y), \\ \dot{y} = -H_x^-(x,y) + \varepsilon q^-(x,y), \end{cases} x < 0, \quad \begin{cases} \dot{x} = H_y^+(x,y) + \varepsilon p^+(x,y), \\ \dot{y} = -H_x^+(x,y) + \varepsilon q^+(x,y), \end{cases} x \geq 0, \tag{1.3}$$

where ε is the same as the one given in (1.1), $H^\pm(x,y),\ p^\pm(x,y)$ and $q^\pm(x,y)$ are C^∞ functions in the variables x and y. Assume that system $(1.3)|_{\varepsilon=0}$ has a family of clockwise periodic orbits intersecting the y-axis with two different points. We denote them by $A_1(h) = (0, a_1(h)), A_2(h) = (0, a_2(h))\ h \in \tilde{\mathcal{I}}$, where $a_1(h) > a_2(h)$. Clearly, the periodic orbits can be rewritten as

$$\widehat{A_1A_2} \cup \widehat{A_2A_1}, \quad h \in \tilde{\mathcal{I}}.$$

Then, by Theorem 1.1 in [13] and Lemma 2.2 in [16], the first order Melnikov function of system (1.3) can be expressed as for $h \in \tilde{\mathcal{I}}$

$$M(h) = \int_{\widehat{A_1A_2}} q^+(x,y)dx - p^+(x,y)dy + \frac{H_y^+(A_1(h))}{H_y^-(A_1(h))} \int_{\widehat{A_2A_1}} q^-(x,y)dx - p^-(x,y)dy, \tag{1.4}$$

which is generalized by [17]. It is easy to see that the function in (1.2) is the function in (1.4) for the smooth case. In other words, formula (1.4) is a generalization of formula (1.2). By introducing multiple small parameters, the papers [18] and [19] generalized the formulas (1.2) and (1.4), respectively, and gave the corresponding applications. Obviously, non-smoothness may occur on multiple lines or even nonlinear curves or surfaces. For system (1.3), the non-smoothness appear on the y-axis. The authors of the paper [20] studied the non-smooth differential system by considering non-smoothness on finitely many nonlinear curves emanating from a vertex. Later, the authors of the paper [21] used the method provided in [20] to study limit cycle bifurcations of a non-smooth differential system. In this paper, motivated by [18–21], we introduce multiple small parameters to study the limit cycle bifurcation of non-smooth differential systems, obtaining some new results.

This paper is organized as follows. In Section 2, we state our main results (Theorems 2.1 and 2.2). The proofs of Theorems 2.1 and 2.2 are given in Sections 3 and 4, respectively.

2. Main results

Consider a differential system of the form

$$\dot{x} = H_y(x,y,\lambda) + \varepsilon p(x,y,\lambda), \quad \dot{y} = -H_x(x,y,\lambda) + \varepsilon q(x,y,\lambda), \tag{2.1}$$

where $0 < \varepsilon \ll \lambda \ll 1$,

$$\big(H(x,y,\lambda), p(x,y,\lambda), q(x,y,\lambda)\big) = \begin{cases} \big(H^{++}(x,y,\lambda), p^{++}(x,y,\lambda), q^{++}(x,y,\lambda)\big),\ x > 0,\ y > 0, \\ \big(H^{-+}(x,y,\lambda), p^{-+}(x,y,\lambda), q^{-+}(x,y,\lambda)\big),\ x < 0,\ y > 0, \\ \big(H^{--}(x,y,\lambda), p^{--}(x,y,\lambda), q^{--}(x,y,\lambda)\big),\ x < 0,\ y < 0, \\ \big(H^{+-}(x,y,\lambda), p^{+-}(x,y,\lambda), q^{+-}(x,y,\lambda)\big),\ x > 0,\ y < 0 \end{cases} \tag{2.2}$$