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Global large-data generalized solutions in a chemotactic movement with rotational flux caused by two stimuli

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ABSTRACT

The main purpose of this paper is to study the global existence of large-data solutions to the following chemotactic model with general rotational sensitivity caused by two stimuli:

$$\begin{cases} u_t = \Delta u - \nabla \cdot \Big(u S_1(x,u,v,w) \nabla v \Big) + \nabla \cdot \Big(u S_2(x,u,v,w) \nabla w \Big), \\ v_t = \Delta v - uv, \\ w_t = \Delta w - uw \end{cases}$$

in a bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary under suitable initial–boundary conditions. Systems of this type arise in mathematical biology as models for the evolution of *Escherichia coli* suspensions in a vertical cylindrical cell by letting the bacteria be uniformly distributed in an oxygen-saturated medium with a glucose concentration step gradient at the mid height of the cell.

For the two-dimensional case, the first author and Li (2016) showed that for suitably regular initial data (u_0, v_0, w_0) fulfilling a smallness condition on the L^∞-norm of v_0 and w_0, the initial–boundary value problem of this system possesses a global bounded classical solution.

In this paper, we will remove such a smallness assumption to show the global existence of generalized solutions with general large initial data by using a new method developed by Winkler (2015). Our result holds in arbitrary dimension $n \geq 1$.

1. Introduction

In nature, motile cells and motile organisms respond to external stimuli by preferential motional organism. As a result, there is a net displacement of the cells toward regions richer in beneficial chemicals (attractants) and away from toxic agents (repellents). Such a mechanism is known as chemotaxis. The main purpose of

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this paper is to study the global existence of large-data solutions to a chemotactic model with two stimuli and general sensitivity. The key physical variables in this model are assumed to be the density of the cell population (denoted by u) and the concentration of the chemoattractant or chemorepellent substances (denoted by v and w). The model is formulated as the following parabolic initial–boundary value problem:

$$\begin{cases} u_t = \Delta u - \nabla \cdot \Big(uS_1(x,u,v,w)\nabla v\Big) + \nabla \cdot \Big(uS_2(x,u,v,w)\nabla w\Big), & x \in \Omega,\ \ t > 0, \\ v_t = \Delta v - uv, & x \in \Omega,\ \ t > 0, \\ w_t = \Delta w - uw, & x \in \Omega,\ \ t > 0, \\ \Big(\nabla u - uS_1(x,u,v,w)\nabla v + uS_2(x,u,v,w)\nabla w\Big) \cdot \nu = 0, \quad \nabla v \cdot \nu = 0, \quad \nabla w \cdot \nu = 0, & x \in \partial\Omega,\ \ t > 0, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad w(x,0) = w_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary, ν represents the outward normal vector field on $\partial\Omega$, and S_1 and S_2 are supposed to be two given parameter functions defined in $\Omega \times [0,\infty)^3$ with values in $\mathbb{R}^{n\times n}$.

A particular example for chemotactic movement caused by two stimuli can be found in Boon–Herpigny [1], where the authors refer to an experiment performed with *Escherichia coli* suspensions in a vertical cylindrical cell by letting the bacteria be uniformly distributed in an oxygen-saturated medium with a glucose concentration step gradient at the mid height of the cell. In this experiment oxygen and glucose are two attractants for the considered *E. coli* population and thus are consumed by the *E. coli* population. Such a consumption mechanism leads to use model (1.1) to take into account the possible competition between two attractants, which would favor chemotaxis toward a specific substrate that is then metabolized preferentially (see also [2] for the more related examples).

Before going into our mathematical analysis, we recall some important progresses on system (1.1) and its variants. A simple model for chemotactic movement was proposed by Keller–Segel [3] and consisted of two equations of the form

$$\begin{cases} u_t = \Delta u - \nabla \cdot \big(\chi u \nabla v\big), \\ v_t = \Delta v - v + u, \end{cases} \tag{1.2}$$

where the chemotactic sensitivity χ means that the chemical signal is an attractive ($\chi > 0$) or repulsive ($\chi < 0$) cue and that the cells move toward or away increasing signal concentration. System (1.2) has rich properties including globally existing solutions, finite time blow-up and spatial pattern formation. For instance, for the case of $\chi = 1$, it has been used in [4] and studied the finite time blow-up of solutions. For instance, it is well-known that the Neumann problem associated with (1.2) in balls $\Omega \subset \mathbb{R}^N$ possesses some solutions blowing up in finite time when either $N \geq 3$, or when $N = 2$ and the total mass of cells $\int_\Omega n_0$ is large [4], while all solutions remain bounded when either $N = 1$, or $N = 2$ and the total mass $\int_\Omega n_0$ is small [5,6]. On the other hand, in some special circumstances, the chemical is repulsive and then the cells may prefer to be away from it. This chemorepulsive phenomenon can be modeled by altering the negative sign afore the chemotactic sensitivity χ in (1.2) to the positive, and, for appropriate regular initial data (without any smallness assumptions), the corresponding chemorepulsion system in $\Omega \subset \mathbb{R}^2$ has a unique smooth classical bounded uniformly in time solution which converges to $\left(\frac{\int_\Omega u_0}{|\Omega|}, \frac{\int_\Omega v_0}{|\Omega|}\right)$ exponentially (see [7]).

In many complex biological processes, cells often interact with a combination of attractive and repulsive signaling chemicals to produce various interesting biological patterns. To describe the competition between the attractive and repulsive signals, Painter–Hillen [8] proposed the following attraction–repulsion