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Walker regime for walls in ferromagnetic nanotubes

Gilles Carbou

CNRS / UNIV PAU & PAYS ADOUR, Laboratoire de Mathématiques et de leurs Applications de Pau, UMR CNRS 5142, Avenue de l'Université - BP 1155, 64013 Pau Cedex, France

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ABSTRACT

Ferromagnetic nanotubes are proposed as an alternative to ferromagnetic nanowires for data-storage applications. In this paper, we consider a two-dimensional model for such devices and we establish the stability of moving walls in the Walker regime when the tube is subject to a small magnetic field.

1. Introduction

Domain walls formation and propagation in ferromagnetic nanowires are intensively studied. Indeed, their possible applications for data recording (see [1]) or in nano-electronics (see [2]) are very promising. Such devices are modeled by a 1d-Landau–Lifschitz equation, and existence and stability of one-wall profiles are established (see [3–6] and the references therein).

In [7], the authors propose to use ferromagnetic nanotubes instead of ferromagnetic nanowires or nano strips in order to deal with domain wall motion in the Walker regime, which is stable and more reliable for applications. In the present work we exhibit a 2d-model for ferromagnetic nanotubes and we study domain wall dynamics in this model for a small applied magnetic field.

Let us recall the 3-dimensional model for a ferromagnetic sample $\mathcal{O} \subset \mathbb{R}^3$. We denote by $(u \cdot v)$ the canonical scalar product of u by v in $\mathbb{R}^3$ and by $|.|$ the associated norm. The canonical basis of $\mathbb{R}^3$ is denoted by (e_1, e_2, e_3) and $\times$ is the usual cross product.

Ferromagnetic materials are characterized by a spontaneous magnetization described by the magnetic moment M defined on $\mathbb{R}^+ \times \mathcal{O}$ and satisfying the saturation constraint

$$|M(\mathrm{t}, x)| = M_s \;\; a.e., \tag{1.1}$$

E-mail address: gilles.carbou@univ-pau.fr.

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where M_s is constant. The magnetic moment satisfies the Landau–Lifschitz equation

$$\frac{\partial M}{\partial \mathrm{t}} = -\gamma M \times H_e - \frac{\alpha\gamma}{M_s} M \times (M \times H_e), \tag{1.2}$$

in which $\gamma > 0$ is the gyromagnetic ratio, $\alpha > 0$ is the damping coefficient, H_e is the effective field given by:

$$H_e = \frac{A}{\mu_0 M_s^2} \Delta M + H_d(M) + H_{app}. \tag{1.3}$$

Here, $A > 0$ is the exchange coefficient, μ_0 is the permeability of the vacuum, H_{app} is the applied magnetic field, and $H_d(M)$ is the demagnetizing field generated by the magnetization M. In the quasi-stationary model, the operator H_d is given by

$$\begin{cases} \mathrm{div}\,(H_d(M) + \overline{M}) = 0, \\ \mathrm{curl}\, H_d(M) = 0, \end{cases} \tag{1.4}$$

where $\overline{M}$ is the extension of M by zero outside $\mathcal{O}$.

The energy associated to a configuration M is given by:

$$\mathcal{E}(M) = \frac{A}{2M_s^2} \int_{\mathcal{O}} |\nabla M|^2 dx + \frac{\mu_0}{2} \int_{\mathbb{R}^3} |H_d(M)|^2 \, dx - \mu_0 \int_{\mathcal{O}} H_a \cdot M \, dx,$$

and we have $H_e = -\frac{1}{\mu_0} \partial_M \mathcal{E}$.

Existence of weak or strong solutions for (1.2) is addressed in several papers (see [8–15]).

We focus now on the case of a thin nanotube of axis $\mathbb{R}e_1$ with circular section. The nanotube is assimilated to the cylinder $\mathbb{R} \times \rho S^1 = \left\{ (\mathrm{x}, \mathrm{y}, \mathrm{z}) \in \mathbb{R}^3, \quad \mathrm{y}^2 + \mathrm{z}^2 = \rho^2 \right\}$. We assume that a magnetic field H_{app} is applied in the direction of the tube axis: $H_{app} = H_a e_1$, $H_a \in \mathbb{R}$. We use the two-dimensional model of ferromagnetic thin film obtained in [16] and [17], in which the demagnetizing field reduces to an anisotropic local term forcing M to be tangent to the thin domain. In the case of our nanotube the demagnetizing field is described by the term $-(M \cdot \mathrm{n})\mathrm{n}$, derived from the limit demagnetizing energy $\frac{\mu_0}{4} \int_{\mathbb{R} \times S^1} |M \cdot \mathrm{n}|^2 d\sigma$, where n is the unit normal vector to the cylinder surface.

In cylindrical coordinates, we write $\mathrm{y} = \rho \cos y$ and $\mathrm{z} = \rho \sin y$, and we obtain the following 2d model:

$$\begin{cases} M : (t, \mathrm{x}, y) \to S^2, \quad 2\pi\text{-periodic in the variable } y, \\ \dfrac{\partial M}{\partial \mathrm{t}} = -\gamma M \times h(M) - \dfrac{\alpha\gamma}{M_s} M \times (M \times h(M)), \\ h(M) = \dfrac{A}{\mu_0 M_s^2} \dfrac{\partial^2 M}{\partial \mathrm{x}^2} + \dfrac{A}{\mu_0 M_s^2 \rho^2} \dfrac{\partial^2 M}{\partial y^2} - (M \cdot \mathrm{n}(y))\mathrm{n}(y) + H_a e_1, \end{cases} \tag{1.5}$$

where the unit normal vector n is given by $\mathrm{n}(y) = \begin{pmatrix} 0 \\ \cos y \\ \sin y \end{pmatrix}$.

We denote $\mathrm{n}^{\perp}(y) = \begin{pmatrix} 0 \\ -\sin y \\ \cos y \end{pmatrix}$. By the rescaling $t = \frac{\gamma A \mathrm{t}}{\mu_0 M_s \rho^2}$ and $x = \frac{\mathrm{x}}{\rho}$, we describe M in the frame $(e_1, \mathrm{n}(y), \mathrm{n}^{\perp}(y))$ writing:

$$M(t, \mathrm{x}, y) = M_s \left(\mathbf{m}_1 \left(\frac{\gamma A \mathrm{t}}{\mu_0 M_s \rho^2}, \frac{\mathrm{x}}{\rho}, y \right) e_1 + \mathbf{m}_2 \left(\frac{\gamma A \mathrm{t}}{\mu_0 M_s \rho^2}, \frac{\mathrm{x}}{\rho}, y \right) \mathrm{n}(y) + \mathbf{m}_3 \left(\frac{\gamma A \mathrm{t}}{\mu_0 M_s \rho^2}, \frac{\mathrm{x}}{\rho}, y \right) \mathrm{n}^{\perp}(y) \right).$$

We obtain that M satisfies (1.5) if and only if $\mathbf{m} = \begin{pmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{pmatrix}$ satisfies

$$\begin{cases} \mathbf{m} : (t, x, y) \to S^2, \quad 2\pi\text{-periodic in the variable } y, \\ \dfrac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}(\mathbf{m}) - \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{h}(\mathbf{m})), \\ \mathbf{h}(\mathbf{m}) = \partial_{xx}\mathbf{m} + \partial_{yy}\mathbf{m} + 2e_1 \times \partial_y \mathbf{m} + \mathbf{m}_1 e_1 - \kappa \mathbf{m}_3 e_3 + h_a e_1, \end{cases} \tag{1.6}$$

where $\kappa = \frac{\mu_0 M_s^2 \rho^2}{A}$ and $h_a = \frac{\mu_0 M_s \rho^2}{A} H_a$.