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Qualitative analysis on positive steady-state solutions for an autocatalysis model with high order☆

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HIGHLIGHTS

- Establish the existence of the steady-state bifurcation at double eigenvalue.
- Obtain some conditions to judge the bifurcation direction.
- An interesting dynamical behavior is observed by numerical simulations.

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ABSTRACT

This paper is concerned with an autocatalysis model with high order under Neumann boundary conditions. Firstly, the stability of the equilibrium is discussed and the effect of diffusion coefficients on Turing instability is described. Next by maximum principle and Poincaré inequality, a priori estimates and some characters of positive steady-state solutions are given. Moreover, the bifurcations at both simple and double eigenvalues are intensively investigated. Using the bifurcation theory, we establish the global structure of the bifurcation from simple eigenvalues and obtain some conditions to determine the bifurcation direction. The techniques of space decomposition and implicit function theorem are adopted to deal with the case of double eigenvalues. Finally, some numerical simulations are shown to supply and supplement the analytical results.

1. Introduction

A single chemical reaction is said to have undergone autocatalysis, or be autocatalytic, if the reaction product is itself the catalyst for that reaction [1]. The rate equations for autocatalytic reactions are fundamentally nonlinear and this nonlinearity can lead to the spontaneous generation of order.

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Nearly all the mechanistic schemes include autocatalytic steps. Consider throughout steps with the overall stoichiometry $U \to V$. We may also take this as written to represent a reaction satisfying the rate law $= ku$. When the same change is catalyzed by a species Y and has a rate satisfying the expression kuy, we may represent this as $U + Y \to V + Y$. Autocatalysis corresponds to catalysis by the product Y itself. Two exemplary cases are quadratic autocatalysis rate kuv and cubic autocatalysis rate kuv^2, see [2,3]. These may be represented by the chemical equations

$$U + V \to 2V \quad \text{and} \quad U + V \to 3V.$$

In the present context, we wish to allow a more general approach in which arbitrary order of autocatalysis can be encompassed. So we consider the prototype chemical reaction scheme based on arbitrary order autocatalator

$$P \to U \quad \text{rate } k_0 p, \qquad U + mV \to (m+1)V \quad \text{rate } kuv^m, \qquad V \to W \quad \text{rate } bv,$$

where p, u and v are the concentrations of the reactant P, U and the autocatalyst V, respectively, and k_0, k, b are the rate constants. Here we assume that the original reactant P is present in large excess and hence its concentration remains constant at its initial value p_0 throughout. Then the equations governing the evolution in concentrations of the reactants in a closed vessel are

$$\begin{cases} u_t - d_1 \Delta u = p_0 - kuv^m, & t > 0, \quad x \in \Omega, \\ v_t - d_2 \Delta v = kuv^m - bv, & t > 0, \quad x \in \Omega, \\ \partial_\nu u = \partial_\nu v = 0, & t > 0, \quad x \in \partial\Omega, \\ u(0,x) = u_0(x) \geq 0, \quad v(0,x) = v_0(x) \geq 0, & x \in \bar{\Omega}, \end{cases} \tag{1.1}$$

where Δ is Laplace operator, d_1, d_2 are the diffusion coefficients, Ω is a bounded domain in $R^N (N \geq 1)$ with smooth boundary $\partial\Omega$ and ν is outward unit normal vector on $\partial\Omega$.

The cubic autocatalysis model, (1.1) with $m = 2$, has received much attentions. Finlayson and Merkin in [4] illustrated the effect of electric field on the spatial structure by linear stability theory and numerical simulation method. Under some conditions, it was shown in [5] that the stable pattern and standing wave of system (1.1) can form. The spatially non-uniform stationary structures, especially bifurcating from the double eigenvalue, was studied in [6] by the use of Lyapunov–Schmidt technique and singularity theory.

Here we point out that m is determined by experiment and not necessary to be a positive integer [7]. A linear stability analysis of planar reaction fronts to transverse perturbations was considered in [8] for a system based on an autocatalytic reaction of general order, and the effect of the order of the autocatalysis on an asymptotic analysis for small wavenumbers was derived. A rigorous proof of the existence of traveling waves for the generalized autocatalytic chemical reaction–diffusion model was given in [9]. Stability and pattern formation in a two-cell coupled arbitrary order of autocatalysis system was investigated in [10], in which Turing bifurcation solutions were obtained by weakly nonlinear theory. Stationary patterns in a two-cell coupled autocatalysis model with arbitrary powers were also discussed in [11], and the non-existence and existence of nontrivial steady-state solutions were established by using implicit function theorem and topological degree theory.

In addition, there are many works on other autocatalysis models. For example, see [12–14] for the Sel'kov model, see [15–17] for the general Brusselator model, see [18–22] for the Lengyel–Epstein model and see [23–25] for a bimolecular model with saturation law. More specifically, the existence and non-existence of non-constant positive steady-state solutions were studied in [13–16,18,19,22,24,26,27]; Hopf bifurcation analyses were performed in [17,20,21,25,28]. In particular, both Hopf bifurcation and steady-state bifurcation were considered in [21,25,29–31].