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Coupling conditions for isothermal gas flow and applications to valves

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ABSTRACT

We consider an isothermal gas flowing through a straight pipe and study the effects of a two-way electronic valve on the flow. The valve is either open or closed according to the pressure gradient and is assumed to act without any time or reaction delay. We first give a notion of coupling solution for the corresponding Riemann problem; then, we highlight and investigate several important properties for the solver, such as coherence, consistence, continuity on initial data and invariant domains. In particular, the notion of coherence introduced here is new and related to commuting behaviors of valves. We provide explicit conditions on the initial data in order that each of these properties is satisfied. The modeling we propose can be easily extended to a very wide class of valves.

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1. Introduction

In this paper we consider a model of gas flow through a pipe in presence of a pressure-regulator valve. We deal with a *plug flow*, which means that the velocity of the gas is constant on any cross-section of the pipe; all friction effects along the walls of the pipe are dropped. To model the flow away from the valve, we use the following equations for conservation of mass and momentum, as done for analogous problems in [1–4]:

$$\begin{cases} \rho_t + (\rho\, v)_x = 0, \\ (\rho\, v)_t + \left(\rho\, v^2 + p(\rho)\right)_x = 0. \end{cases} \tag{1}$$

Here $t > 0$ is the time and $x \in \mathbb{R}$ is the space position along the pipe. The state variables are ρ, the *mass density* of the gas and v, the *velocity*; we denote by $q \doteq \rho\, v$ the *linear momentum*. Since variations of temperature are not significant in most real situations of gas flows in pipes, we focus on the *isothermal* case

$$p(\rho) \doteq a^2 \rho, \tag{2}$$

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for a constant $a > 0$ that gives the *sound speed*. We emphasize that the flow can occur in either directions along the pipe; it can be either subsonic or supersonic. Usually, an hydraulic system is completed by compressors [4–8] and valves [9,10]. In this paper we focus on the case of a valve.

Indeed, there are several different kinds of valves, but their common feature consists in regulating the flow. Opening and closing can be partial and may depend either on the flow, or on the pressure, or even on a combination of both. Moreover, a valve may let the gas flow in one direction only or in either. The simplest and most natural problem for system (1) in presence of a valve is clearly the Riemann problem, where the valve induces a substantial modification in the solutions with respect to the free-flow case. However, proposing a Riemann solver that includes the mechanical action of a valve is only the first step toward a good description of the flow for positive times: some natural properties, both from the physical and mathematical point of view, have to be investigated. Such properties are coherence, consistence and continuity with respect to the initial data; at the end, if possible, invariant domains should be properly established. This is the main issue of this paper.

In Section 2 we rigorously define the notions mentioned above; they are stated in the case of system (1) but can be readily extended to any "nonstandard" coupling Riemann solver. A very short account on the Lax curves of (1) is then given as well as the definition of the standard Riemann solver for this system. This material is very well known [11], but it is so heavily exploited in the following that any comprehension would be hindered without these details.

Section 3 introduces a "Riemann solver" when an interface condition, such as that given by a valve, is present. Some general results are then given and few simple models of valves (see [12, §2], [9, (6)] or [10, § 4.3.2, § 4.3.3, (1)–(4) page 51]) are provided. In this modeling, we do not take into consideration the flow inside the valve but simply its effects. The framework is that of conservation laws with point constraints, which has so far been developed only for vehicular and pedestrian flows, see [13,14] and the references therein.

Section 4 contains our main results, which are collected in Theorem 4.1. They concern the coherence, consistence, continuity with respect to the initial data and invariant domains in a very special case, namely that of a pressure-relief valve. They can be understood as a first step in the direction of proving a general existence theorem for initial data with bounded variation. Some technical proofs are collected in Section 5. The final Section 6 resumes our conclusions.

2. The gas flow through a pipe

In this introductory section we provide some information about system (1), in particular as far as the geometry of the Lax curves is concerned.

2.1. The system and basic definitions

Under (2), system (1) can be written in the conservative (ρ, q)-coordinates as

$$\begin{cases} \rho_t + q_x = 0, \\ q_t + \left(\dfrac{q^2}{\rho} + a^2\rho\right)_x = 0. \end{cases} \tag{3}$$

We usually refer to the expression (3) of the equations and denote $u \doteq (\rho, q)$. We assume that the gas fills the whole pipe and then u takes values in $\Omega \doteq \{(\rho, q) \in \mathbb{R}^2 : \rho > 0\}$. A state (ρ, q) is called *subsonic* if $|q/\rho| < a$ and *supersonic* if $|q/\rho| > a$; the half lines $q = \pm a\,\rho$, $\rho > 0$, are *sonic lines*.

The Riemann problem for (3) is the Cauchy problem with initial condition

$$u(0, x) = \begin{cases} u_\ell & \text{if } x < 0, \\ u_r & \text{if } x > 0, \end{cases} \tag{4}$$

$u_\ell, u_r \in \Omega$ being given constants.

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