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Bifurcation of limit cycles by perturbing piecewise smooth integrable non-Hamiltonian systems

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ABSTRACT

This paper deals with bifurcation of limit cycles for piecewise smooth integrable non-Hamiltonian systems. We derive the first order Melnikov function, which plays an important role in the study of the number of limit cycles bifurcated from the periodic annulus of a center. As an application, we consider a class of cubic isochronous centers, which has a non-rational first integral. Using the first order Melnikov function, we obtain the sharp upper bound of the number of limit cycles which bifurcate from the periodic annulus of the center under piecewise smooth polynomial perturbations.

1. Introduction and statement of main results

One of the main topics in the qualitative theory of planar differential systems is to determine the numbers and distributions of limit cycles. The restriction of this problem to polynomial differential systems is the well-known Hilbert's 16th problem, see [1]. Since Hilbert's 16th problem turns out to be a strongly difficult one, many mathematicians began to study some special kinds of polynomial systems, such as the weak Hilbert's 16th problem [2], Liénard system [3–5], quadratic system [6,7] and cubic system [8].

Consider the perturbed planar smooth Hamiltonian system

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = H_y(x,y) + \varepsilon f(x,y), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -H_x(x,y) + \varepsilon g(x,y), \end{cases} \tag{1}$$

where $f(x,y)$ and $g(x,y)$ are analytic functions with respect to x, y. We suppose that the unperturbed system $(1)|_{\varepsilon=0}$ has a family of periodic orbits $L_h \subset \{(x,y) : H(x,y) = h \in J = (\alpha,\beta)\}$.

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A classic way to generate limit cycles is by perturbing a system which has a center via Poincaré bifurcation, in such a way that limit cycles bifurcate in the perturbed system from the periodic annulus of the center for the unperturbed system. In order to study the maximum number of limit cycles of system (1) which bifurcate from the periodic annulus of the center, it is necessary to study the number of zeros of the first order Melnikov function, also known as Abelian integral, i.e.,

$$M_1(h) = \oint_{L_h} g(x, y)\mathrm{d}x - f(x, y)\mathrm{d}y. \tag{2}$$

In the case that (1) is a polynomial system, the study of the upper bound of the number of zeros of (2) is the so called weak Hilbert's 16th problem, see [2]. Many works have been done on limit cycles by perturbing Hamiltonian system with a family of periodic orbits, see the survey paper [2].

For the perturbed planar smooth integrable non-Hamiltonian system

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{H_y(x, y)}{R(x, y)} + \varepsilon f(x, y), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{H_x(x, y)}{R(x, y)} + \varepsilon g(x, y), \end{cases} \tag{3}$$

with integrating factor $R(x, y)$, the first order Melnikov function can be expressed as

$$\overline{M}_1(h) = \oint_{L_h} R(x, y)g(x, y)\mathrm{d}x - R(x, y)f(x, y)\mathrm{d}y. \tag{4}$$

There are some works on limit cycles by perturbing integrable non-Hamiltonian system, see for instance [9,10].

In recent years, stimulated by the discontinuous phenomena in the real world, there has been considerable interest in studying the bifurcation of piecewise smooth differential system, see for instance [11] and the references therein. There are many authors generalizing the Hilbert's 16 problem to the piecewise smooth case, that is to say, they consider the limit cycles for the piecewise smooth Hamiltonian system, see for instance [12–17].

The authors in the paper [18] extend the averaging method to piecewise smooth system. In order to apply the averaging method, a transformation of the original system into the standard form must be done. Generally speaking, if the system is integrable but no-Hamiltonian, then the translation is difficult.

In the paper [15], the authors consider the perturbed planar piecewise smooth Hamiltonian systems

$$\begin{pmatrix} \dfrac{\mathrm{d}x}{\mathrm{d}t} \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} \end{pmatrix} = \begin{cases} \begin{pmatrix} H_y^+(x, y) + \varepsilon f^+(x, y) \\ -H_x^+(x, y) + \varepsilon g^+(x, y) \end{pmatrix}, & x > 0, \\ \begin{pmatrix} H_y^-(x, y) + \varepsilon f^-(x, y) \\ -H_x^-(x, y) + \varepsilon g^-(x, y) \end{pmatrix}, & x \leqslant 0, \end{cases} \tag{5}$$

where $f^\pm(x, y), g^\pm(x, y)$ and $H^\pm(x, y)$ are analytic functions with respect to x and y. The switching manifold $x = 0$ divides $\mathbb{R}^2$ into two regions, where the systems are smooth in each region. We call system (5) with $x > 0$ the right subsystem and $x < 0$ the left subsystem respectively.

Suppose that the unperturbed system $(5)|_{\varepsilon=0}$ has a family of periodic orbits around the origin and satisfy the following two assumptions:

Assumption (I). There exist an open interval $J = (\alpha, \beta)$, and two points $A(h) = (0, a(h))$, $A_1(h) = (0, a_1(h))$, where $a(h) \neq a_1(h)$. For $h \in J$, we have

$$H^+(A(h)) = H^+(A_1(h)) = h, \qquad H^-(A(h)) = H^-(A_1(h)). \tag{6}$$