Nonlinear Analysis: Real World Applications 34 (2017) 159–171

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

On the symmetry of equatorial travelling water waves with constant vorticity and stagnation points

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ARTICLE INFO

Article history:
Received 29 June 2016
Received in revised form 22 August 2016
Accepted 27 August 2016

Keywords:
Symmetry
Free-boundary
Vorticity
Stagnation points
f-plane approximation

ABSTRACT

The aim of this paper is to prove that equatorial travelling water waves at the surface of water flows with constant vorticity are symmetric, provided they have a profile that is monotonic between crests and troughs and that there are no stagnation points in the subsurface region.

1. Introduction

The dynamics of water flows throughout the equatorial extent of the Pacific Ocean (about 13.000 km) is very complicated. One aspect is the presence of the Equatorial Undercurrent (EUC), cf. [1], which is characterized by vertical flow variations near the surface (at great depths the water is motionless). While, due to winds that blow westward, the surface water flow is directed westward, the flow reverses at a depth of several tens of metres. A further feature of waves in the equatorial region is the smallness of the variation in latitude of the EUC. This implies that the variations of the Coriolis parameter can be neglected and one can use the f-plane approximation (see [1]).

In the last decade there have appeared several papers involving rotational water waves on topics such as existence of travelling waves [2–4], regularity of the free surface and of the stream lines [5–11], symmetry [12–17] and stability [18,19]. Nevertheless, results which take into account the Earth's rotation and incorporate it in the equations of motions are very recent [20–24,15,25–31] and concern gravity and capillary–gravity water waves. In this paper we consider equatorial geophysical water waves, where we allow for the existence of stagnation points in a flow of constant non-vanishing vorticity; note that internal stagnation points are

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http://dx.doi.org/10.1016/j.nonrwa.2016.08.010
1468-1218/

not possible in irrotational flows (see the discussion in [32]). Note that the rigorous treatment of water flows with stagnation points is relatively recent and concerns gravity and capillary–gravity water waves (see [3,11] for the existence of a local bifurcation curve of solutions and [33] for the global counterpart). Closely related to the existence of stagnation points is the appearance of critical layers, see [34,35].

In [17] the author shows that gravity waves which are monotonic between crests and troughs with constant vorticity and allowing for stagnation points are symmetric. This paper aims to extend the proofs of [12,17] to include Coriolis effects, proving the symmetry property of equatorial waves which are monotonic between crests and troughs. We employ sharp maximum principles for elliptic partial differential operators for the proof.

2. The governing equations

Firstly, we recall the governing equations in the f- plane approximation of equatorial water waves. These are the Euler's equations and the equation of mass conservation. We consider water to be an inviscid, incompressible fluid [32], which means that its density ρ is taken to be a constant.

Choose Cartesian coordinates (x, y) so that the x-axis points due east, the y-axis points upwards and the origin lies on the flat bed. The free surface of the water is given by $y = \eta(x,t)$, $\boldsymbol{u}(\boldsymbol{x}, t) = (u(x, y, t), v(x, y, t))$ is the velocity of the flow and $P(x, y, t)$ is the pressure.

The equation of mass conservation is

$$u_x + v_y = 0.$$

Euler's equations in this setting are

$$\begin{cases} u_t + uu_x + vu_y + 2\omega v = -\dfrac{P_x}{\rho} \\ v_t + uv_x + vv_y - 2\omega u = -\dfrac{P_y}{\rho} - g, \end{cases} \tag{1}$$

where $\omega = 73 \cdot 10^{-6}$ rad/s is the Earth's constant rotational speed and g is the gravitational acceleration. We consider the Earth to be a perfect sphere of radius 6371 km which rotates about the y-axis.

The boundary conditions which we impose on the system are of two kinds: the *dynamic* and the *kinematic* boundary conditions. The dynamic boundary condition is

$$P = P_{atm} \quad \text{on } y = \eta(x,t),$$

where P_{atm} is the constant atmospheric pressure. This boundary condition separates the motion of the water from that of the air above it. The kinematic boundary conditions are

$$v = \eta_t + u\eta_x \quad \text{on } y = \eta(x,t) \tag{2}$$

and

$$v = 0 \quad \text{on } y = 0. \tag{3}$$

They ensure that once a particle is on the free surface at time t_0 it will stay there at $t > t_0$ and that water cannot penetrate the rigid bed. On the derivation of Euler's equation and the boundary conditions see [32].

An essential property of a fluid flow is the *vorticity*

$$\nabla \times \boldsymbol{u},$$

which measures the local spin or rotation of a fluid element (thus producing a swirling motion). For two-dimensional flows $\boldsymbol{u} = (u, v, 0)$. In this paper we consider the vorticity to be a constant different from zero and denote it by $\gamma \equiv const \neq 0$.