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The Cauchy problem for a Bardina–Oldroyd model to the incompressible viscoelastic flow

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ABSTRACT

In this paper, we study the Cauchy problem for a regularized viscoelastic fluid model in space dimension two, the Bardina–Oldroyd model, which is inspired by the simplified Bardina model for the turbulent flows of fluids, introduced by Cao et al. (2006). In particular, we obtain the local existence of smooth solutions to this model via the contraction mapping principle. Furthermore, we prove the global existence of smooth solutions to this system.

1. Introduction

In this article, our study is concerned with the following incompressible Oldroyd model describing the incompressible non-Newtonian fluid:

$$\begin{cases} \partial_t u + (u \cdot \nabla)u - \mu\Delta u + \nabla P = \nabla \cdot (FF^T), \\ \partial_t F + (u \cdot \nabla)F = \nabla u F, \\ \nabla \cdot u = 0, \quad x \in \mathbb{R}^n, \ n = 2, 3, \\ u(0, x) = u_0, \end{cases} \tag{1.1}$$

where $u(t, x)$ denotes the fluid velocity vector field, $P = P(t, x)$ is the scalar pressure, $F = F(t, x) \in \mathbb{R}^n \times \mathbb{R}^n$ the deformation tensor, $\mu > 0$ is the constant kinematic viscosity, while u_0 is the given initial velocity with $\nabla \cdot u_0 = 0$.

There has been a lot of work on the existence theory of Oldroyd model (1.1). In particular, Lin–Liu–Zhang [1] proved global existence in the two-dimensional case by introducing an auxiliary vector field to replace the transport variable F, while Lei–Zhou [2] proved the same results via the incompressible limit working directly on the deformation tensor F. It is worthy mentioning that Lei–Liu–Zhou [3] proved

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the existence of both local and global smooth solutions to the Cauchy problem in the whole space and the periodic problem in the n-dimensional torus ($n = 2, 3$) in the case of near equilibrium initial data, while Lin–Zhang [4] established the global well-posedness of the initial–boundary value problem of the viscoelastic fluid system of the Oldroyd model with Dirichlet conditions. More discussions can be found in [5–12].

Viscoelastic materials include a wide range of fluids with elastic properties, as well as solids with fluid properties. To introduce our model, we need to give the standard description of general mechanical evolutions to introduce some notations and definitions and simplify the system (1.1). In the context of hydrodynamics, the basic variable is the particle trajectory $x(t, X)$, where X is the original labeling (Lagrangian coordinate) of the particle and referred to as the material coordinate, while x is the current (Eulerian) coordinate and referred to as the reference coordinate. For a given velocity field $u(t, x)$, the flow map $x(t, X)$ is defined by the following ordinary differential equation:

$$\frac{\partial x(t,X)}{\partial t} = u(t, x(t, X)), \quad x(0, X) = X.$$

The deformation tensor is then defined by $\tilde{F}(t, X) = \frac{\partial x(t,X)}{\partial X}$. In the Eulerian coordinate, the corresponding deformation tensor $F(t, x)$ is defined as $F(t, x(t, X)) = \tilde{F}(t, X)$. Using the chain rule, one can see that $F(t, x)$ satisfies the following transport equation, i.e. the second equation of (1.1):

$$\partial_t F + (u \cdot \nabla)F = \nabla u F.$$

If $\nabla \cdot F^T(0, x) = 0$, then we get from the second equations of (1.1):

$$\partial_t(\nabla \cdot F^T) + (u \cdot \nabla)(\nabla \cdot F^T) = 0.$$

Therefore, if $\nabla \cdot F^T(0, x) = 0$, it will remain so for later times, namely, $\nabla \cdot F^T = 0$ for any time $t > 0$. In what follows, we will make this assumption. Denote $F_k = Fe_k$ as the columns of F, then $\nabla \cdot (FF^T) = \sum_{k=1}^n (F_k \cdot \nabla)F_k$ by the fact $\nabla \cdot F_k = 0$. So the system (1.1) can be rewritten equivalently as

$$\begin{cases} \partial_t u + (u \cdot \nabla)u - \mu\Delta u + \nabla P = \sum_{k=1}^n (F_k \cdot \nabla)F_k, \\ \partial_t F_k + (u \cdot \nabla)F_k = (F_k \cdot \nabla)u, \\ \nabla \cdot u = \nabla \cdot F_k = 0, \end{cases} \tag{1.2}$$

with $k = 1, \ldots, n$, and $u(0, x) = u_0, F_k(0, x) = F_{k,0}$.

It is known that the global regularity problem for the 3D Navier–Stokes equations (NSE) is one of the most challenging outstanding problems in nonlinear analysis. The main difficulty lies in controlling certain norms of vorticity. More specifically, the vorticity stretching term in the 3D vorticity equation forms the main obstacle to achieving this control. Numerical solution of the Navier–Stokes equations for problems of engineering and geophysical relevance is not possible at present even on the most powerful computers [13,14]. In turbulent fluid flows, current scientific methods and tools are unable to compute the turbulent behavior of three-dimensional fluids analytically or via direct numerical simulation due to the large range of scales of motion that need to be resolved when the Reynolds number is high. To overcome this difficulty, much effort is being made to produce reliable turbulence models which parameterize the effect of the active small scales in terms of the large scales. The simplified Bardina model, introduced by Cao et al. [15], enjoys the most important property that it compares successfully with empirical data from turbulent channel and pipe flows, for a wide range of Reynolds numbers. So it is proved that it is a good sub-grid scale large-eddy simulation model of turbulence, just as the viscous Camassa–Holm equations (also known as the Lagrangian-averaged Navier–Stokes-α (LANS-α) model) (more discussions can be found in [16–21]). Based on the success of the simplified Bardina model in producing solutions in excellent agreement with empirical data, it is natural to lead us to consider such a kind of regularization also for the Oldroyd model.