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The motion of the rigid body in the viscous fluid including collisions. Global solvability result

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ABSTRACT

We shall consider the problem of the motion of a rigid body in an incompressible viscous fluid filling a bounded domain. This problem was studied by several authors. They mostly considered classical non-slip boundary conditions, which gave them very paradoxical result of no collisions of the body with the boundary of the domain. Only recently there are results when the Navier type of boundary is considered.

In our paper we shall consider the Navier condition on the boundary of the body and the non-slip condition on the boundary of the domain. This case admits collisions of the body with the boundary of the domain. We shall prove the global existence of weak solution of the problem.

1. Introduction

The problem of the motion of one or several rigid bodies in a viscous fluid filling a bounded domain was studied by several authors [1–4]. In [1] authors have shown the global-in-time existence of weak solutions provided that the rigid body did not touch the boundary. In [2] it has been shown that in the absence of collisions, solutions have existed for all time in 2-dimension, whereas in 3-dimension the lifespan of solutions is infinite only for small enough data. In [3] the global existence of weak solutions away from collisions was proved. Hoffmann, Starovoitov [4] have proved the global solvability result under the assumption that the boundaries of the body and of the container are curves of the class C^2. In this case they have shown that the body hits the wall with zero speed and did not move during touching.

In all mentioned articles a non-slip boundary condition has been considered on the boundaries of the bodies and of the domain. This condition gives a very paradoxical result of no collisions between the bodies and the boundary of the domain, see works of Hesla [5], Hillairet [6]. This result was extended to the three dimensional situation by Hillairet, Takahashi [7].

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Starovoitov [8] (see also San Martin et al. [9]) has shown that collisions, if any, must occur with zero relative *translational* velocity as soon as the boundaries of the rigid bodies are enough smooth and the gradient of the underlying velocity field is square integrable — a hypothesis satisfied by any Newtonian fluid flow of finite energy. Hence the possibility or impossibility of collisions in viscous fluids is related to the properties of the velocity gradient and to the regularity of boundaries. Indeed Starovoitov [8] has proved that collisions of two or more rigid bodies are *impossible* in $\mathbb{R}^3$ if:

- the physical domain $\Omega \subset \mathbb{R}^3$ as well as the rigid objects in its interior have boundaries of class $C^{1,1}$ and
- the pth power of the velocity gradient is integrable with $p \geqslant 4$.

Applying this result it has been demonstrated that in the case of very viscous fluids (for instance non-Newtonian fluids) and if the initial distance between the bodies is positive, then the distance between these bodies continues to be positive in a finite time (see Feireisl et al. [10]). Also we would like to mention the work of Gérard-Varet, Hillairet [11], where the existence of the strong solution has been shown under the regularity of boundaries $C^{1,\alpha}$, and proved that the collisions are possible in a finite time if and only if $\alpha < 1/2$. Let us mention also the possibility of the so-called grazing collisions [12]. The above mentioned results have demonstrated that we need a more accurate model, describing the motion of the rigid bodies in the viscous incompressible fluid.

Our article is devoted to the problem of the motion of the rigid body in the viscous fluid when a slippage is allowed at boundaries. The slippage is prescribed by the Navier boundary condition, having only the continuity of the velocity just in the normal component. To our knowledge the first solvability result was done by Neustupa, Penel [13,14], in a particular situation, where they have considered a prescribed collision of a ball with a wall and the slippage was allowed on both boundaries. Their pioneer result shown that the slip boundary condition cleans the no-collision paradox. Recently Gérard-Varet, Hillairet [15] have proved a local-in-time existence result: up to collisions. See also the work of Planas, Sueur [16] related with a single body moved in the whole space $\mathbb{R}^3$.

Gérard-Varet et al. [17] have considered the free fall of a sphere above a wall, that is when the boundaries are C^∞-smooth, in a viscous incompressible fluid in two different situations:

- *Mixed case:* the Navier boundary condition is prescribed on the boundary of the body and the non-slip boundary condition — on the boundary of the domain;
- *Slip case:* the Navier boundary conditions are prescribed on both boundaries as of the body and of the domain.

The result of them is interesting, saying that in the *Mixed case* the sphere never touches the wall and in the *Slip case* the sphere reaches the wall during a finite time period.

The aim of our article is to consider the *Mixed case*, proving the global solvability result, presented in Theorem 2.1, even if the collisions of the body with the boundary of the domain occur in a finite time under a lower regularity of the boundaries of the body and of the domain than was assumed in [17]. The investigation of the *Slip case* is our future work.

We shall investigate the motion of a rigid body inside of a viscous incompressible fluid. The fluid and the body occupy a bounded domain $\Omega \subset \mathbb{R}^N$ ($N = 2$ or 3). Let the body be an open connected set $S_0 \subset \Omega$ at the initial time $t = 0$. The fluid fills the domain $F_0 = \Omega \backslash \overline{S_0}$ at $t = 0$.

The Cartesian coordinates $\mathbf{y}$ of points of the body at $t = 0$ are called the Lagrangian coordinates. The motion of any material point $\mathbf{y} = (y_1, \ldots, y_N)^T \in S_0$ is described by two functions

$$t \to \mathbf{q}(t) \in \mathbb{R}^N \quad \text{and} \quad t \mapsto \mathbb{Q}(t) \in SO(N) \quad \text{for } t \in [0, T],$$