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A local existence of viscous self-organized hydrodynamic model

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ABSTRACT

We establish the local existence result of self-organized hydrodynamic model with viscosity in three dimension case, which was proposed in Degond–Liu–Motsch–Panferov's previous work and was left as one open case. Our strategy relies on the suitable symmetrizing the SOH model in spherical coordinates, which enables us to control the commutator arising in the pressure contribution terms during the process of energy estimate.

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1. Introduction

The self-propelled particle systems arising from animal societies interacting through alignment, such as bird flocks, fish schools, insect swarms, etc., have been widely studied in recent years, and there has been an intense amount of literature on the modeling of collective motion and self-organization from a viewpoint of mathematics. Here we refer to [1–3] for the recent reviews on this subject. Among these models, the Vicsek model, proposed by Vicsek et al. [4] to describe the dynamics on orientation for self-propelled particles, has received much attention for its structural simplicity but indicating a wide range of qualitative features. This Vicsek model is discrete in time, each agent as particle moves with the same constant speed and tries to align its velocity orientation with the local average orientation of its neighbors up to some noise. A time-continuous version as a variant of Vicsek model has been proposed by Degond–Motsch [5], in which the kinetic formation and its hydrodynamic limit are also available. Bolley–Cañizo–Carrillo justified in [6] the rigorous derivation of the kinetic model from the time-continuous model.

We note that in [5], the authors performed a formally asymptotic analysis of the kinetic Vicsek model to derive the hydrodynamic limit, which referred to later as the Self-Organized Hydrodynamics (SOH), is a

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balance system for the density and average velocity orientation (or, polarization vector). Frouvelle considered in [7] the cases that the relaxation parameter depends on the local density and the influence of a vision angle which requires some anisotropy in the observation kernel, and the author also provided a general method to get asymptotic expansions of the coefficients.

Some modified SOH models have also been investigated by Degond–Liu–Motsch–Panferov [8], based on introducing the attraction–repulsion force and different scaling assumptions about the size of the sensing region associated to a higher level of non-locality. The attraction–repulsion force will lead to either an additional pressure term or a capillary (or, Korteweg) force term in the hydrodynamic limit (SOH) model, depending on the scalings, while the non-locality will result at the macroscopic level in the appearance of higher order derivative terms, for instance the viscosity terms. Besides the derivation of the macroscopic models, the authors proved in [8] the local-in-time existence results in 2D for the viscous model and in 3D only for the inviscid model. Both proofs are based on a suitable symmetrization of the SOH system and on the energy estimates. Frouvelle–Liu [9] showed the well-posedness result under a nonnegative initial data in any Sobolev spaces for the kinetic self-organized model in the spatially homogeneous case, which could be also applied to the general homogeneous model, as stated in [10]. The passage from the kinetic model to SOH model was rigorously justified in [11] by the Hilbert expansion approach in kinetic theory. We prove in this present work the local existence for the viscous model in 3D case (see Theorem 1.2), which was proposed in [8] as an open case and has not been studied before. Based on the local existence result for strong solutions, one can obtain the hydrodynamic limit from the kinetic model to the SOH model, in the same way as in [11].

At the end, it should be pointed out that many related aspects of the Vicsek type model have also been studied, such as numerical simulations, phase transition and derivation of macroscopic models mentioned before. We refer to [12,9,10] and the recent review work [2], but an exhaustive bibliography is out of reach.

1.1. Self-organized Hydrodynamic (SOH) model

We consider in this present paper the Cauchy problem of self-organized hydrodynamic (SOH) model introduced in [8],

$$\begin{cases} \partial_t \rho + \mathrm{div}(\rho \Omega) = 0, \\ \rho(\partial_t \Omega + c\Omega \cdot \nabla_x \Omega) + P_{\Omega^\perp} \nabla_x p(\rho) = \mu P_{\Omega^\perp} \Delta_x (\rho \Omega), \\ |\Omega| = 1, \\ \rho(t, x)|_{t=0} = \rho^{in}(x) \geq c_{0>0}, \qquad \Omega(t, x)|_{t=0} = \Omega^{in}(x), \quad |\Omega^{in}| = 1, \end{cases} \tag{1.1}$$

where the unknown $\rho = \rho(t, x) \geq 0$ and $\Omega = \Omega(t, x)$ stand for the density and average velocity with respect to time variable $t \geq 0$ and spatial variables $x \in \mathbb{T}^3 = [0, 1]^3$. Note that the condition $|\Omega| = 1$ implies that Ω denotes indeed the velocity orientation. The coefficients $c \in \mathbb{R}$ and $\mu \geq 0$ are given constants, the pressure $p(\rho)$ is smooth satisfying $p'(p) > 0$. And moreover, we denote by $P_{\Omega^\perp} = \mathrm{Id} - \Omega \otimes \Omega$ the projection matrix onto the normal plane to Ω.

The SOH system evidently bears many similarities with the isentropic compressible Navier–Stokes (NS) system. Both two models can be viewed as nonlinear systems of first order equations perturbed by diffusion. Both nonlinear first order parts on the left-hand sides of Eqs. (1.1) $_1$ and (1.1) $_2$ are hyperbolic. We emphasize that this structure admits a symmetrizer, which allows the development of energy estimates in our proof below.

One important difference between the SOH and NS models is that the SOH system is a non-conservative system due to the fact that the multiplication of projection operator $P_{\Omega^\perp} = \mathrm{Id} - \Omega \otimes \Omega$ is a non-trivial function of Ω. Generally speaking, the coefficient c is not equal to 1, see [2,5].

The second important difference is that, the SOH system obeys the geometric constraint $|\Omega| = 1$ which requires the velocity Ω to be of unit norm, while the NS model does not involve such kind of constraints.

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