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Boundedness properties of very weak solutions to a fully parabolic chemotaxis-system with logistic source



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ABSTRACT

In this paper we study the chemotaxis-system

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + g(u) & x \in \Omega, t > 0, \\ v_t = \Delta v - v + u & x \in \Omega, t > 0, \end{cases}$$

defined in a convex smooth and bounded domain  $\Omega$  of  $\mathbb{R}^3$ , with  $\chi > 0$  and endowed with homogeneous Neumann boundary conditions. The source  $g$  behaves similarly to the logistic function and verifies  $g(s) \leq a - bs^\alpha$ , for  $s \geq 0$ , with  $a \geq 0$ ,  $b > 0$  and  $\alpha > 1$ . In line with Vigliani (2016), where for  $\alpha \in (\frac{5}{3}, 2)$  the global existence of very weak solutions  $(u, v)$  to the system is shown for any nonnegative initial data  $(u_0, v_0) \in C^0(\bar{\Omega}) \times C^2(\bar{\Omega})$  and under zero-flux boundary condition on  $v_0$ , we prove that no chemotactic collapse for these solutions may present over time. More precisely, we establish that if the ratio  $\frac{a}{b}$  does not exceed a certain value and for  $\frac{9}{5} < p < \alpha < 2$  the initial data are such that  $\|u_0\|_{L^p(\Omega)}$  and  $\|\nabla v_0\|_{L^4(\Omega)}$  are small enough, then  $(u, v)$  is uniformly-in-time bounded.

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1. Introduction and motivations

The classical model of population dynamics, originally formulated by Pierre-François Verhulst in 1838 (see [1]), establishes that the self-limiting growth of a biological population size at a certain time,  $P(t)$ , is described through the so-called *logistic equation*  $\frac{dP}{dt} = RP(1 - \frac{P}{K})$ , where the constant  $R$  defines the growth rate and  $K$  is the carrying capacity of the species, which is also associated to the death rate of the same species. This pioneer formulation does not distinguish the portions of the space where the population is more or less distributed, neither the presence of further factors which induce the migration of the population from one zone to another.

On the contrary, the term *chemotaxis* is exactly employed to explain the movement of cells occupying a space, which are stimulated by a chemical signal produced by a substance therein inhomogeneously

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distributed. Studies on the *Escherichia coli* show that such a bacterium directs its motion and changes continuously and randomly its direction precisely depending on the intensity of the chemical gradient stimulus.

In 1970 Keller and Segel proposed a mathematical model for the description of such a phenomenon; it is defined by two parabolic differential equations (see [2]) and the corresponding formulation is

$$\begin{cases} u_t = \nabla \cdot (\rho \nabla u - \chi u \nabla v), \\ v_t = \Delta v - \kappa v + u. \end{cases} \quad (1)$$

In system (1), the distribution of the cells and the concentration of chemical signal in a point  $x$  of the space and at an instant  $t$  of time are, respectively, identified with the functions  $u = u(x, t)$  and  $v = v(x, t)$ . The parameters  $\rho, \chi$  and  $\kappa$  are positive constants. The chemoattractant spreads diffusively, decays with rate  $\kappa$  and is also produced by the bacteria with rate 1. The bacteria diffuse with mobility  $\rho$  and drift in the direction of the gradient of concentration of the chemoattractant with velocity  $\chi|\nabla v|$ ;  $\chi$  is called chemosensitivity. Hence, once the initial cells distribution and chemical concentration (that is  $u_0(x) = u(x, 0)$  and  $v_0(x) = v(x, 0)$ ) are given, under zero-flux boundary conditions on both  $u$  and  $v$ , the previous problem describes the dynamic of a cells population in response to a chemical substance which share a totally insulated domain.

The phenomenon of the *chemotactic collapse* consists of an uncontrolled gathering of cells in proximity of some zones of the space. In fact, real experiments and observations show that the aforementioned movement may eventually lead to such aggregation processes; by the mathematical point of view, the occurrence of such a situation implies that possibly  $u$ , in a certain instant (blow-up time), becomes unbounded in one or more points of its domain. It is known that, in a one-dimensional domain, all the solutions of (1) are global and uniformly bounded in time (see [3]), while that in the  $n$ -dimensional context, with  $n \geq 2$ , unbounded solutions to the same problem have been detected (see, for instance, [4–6]).

In line with the chemotactic scenario, in [7] for radial solutions and in [8] for non-radial, the authors prove that under suitable assumptions the bacteria concentration blows up in finite time, for certain domains of  $\mathbb{R}^2$  and in the cases in which the second differential equation of (1) is replaced by  $0 = \Delta v - v + u$  (parabolic–elliptic case). Moreover, for the classical parabolic–parabolic (or fully parabolic) case, estimates from below and numerical computations for the blow-up time of unbounded solutions to (1) are derived in [9,10], respectively. In [11] the same question concerning lower bounds for the blow-up time of these solutions is addressed under the hypothesis that the parameters modeling the chemotaxis phenomena are time dependent functions.

Furthermore, important and recent results concerning the existence of both bounded or unbounded solutions to chemotaxis-systems have been also attained for a more general class of problems, where the first equation of (1) reads  $u_t = \nabla \cdot (S(u)\nabla u) - \nabla \cdot (T(u)\nabla v)$ . Precisely, the asymptotic behavior of the ratio between the chemosensitivity  $T(u)$  and the mobility  $S(u)$ , determines, also in terms of the space dimension, the main properties (boundedness or unboundedness) of the corresponding solutions. We refer, for instance, to [12,13] for the parabolic–elliptic case and to [14–19] for the parabolic–parabolic one.

Unlike the opening Verhulst model, the original Keller–Segel formulation does not directly consider that the temporal evolution of a cell distribution may be perturbed by the proliferation and the death of the cells themselves; this is mathematically expressed by adding to the first equation of system (1) a linear combination of power functions depending on  $u$  and, possibly, on  $|\nabla u|$ ; specifically, in such models, the contribution of any term with positive (negative) coefficient corresponds to a birth (death) rate (see some details in [20–22] for pure chemotaxis-systems, but also in [23] for weakly coupled systems).

Exactly in view of its importance as a first approach towards the model of self-organizing behavior of cells populations, this investigation focuses on fully parabolic chemotaxis-systems which are complemented by *logistic-type* effects. Such generalization, which is lately challenging many researchers, is not automatic but rather makes the analysis more complex with respect to that in the original model. For instance, to the best of our knowledge, these are the most recent results in this regard. Under Neumann boundary conditions:

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