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Dynamics in a diffusive predator–prey system with a constant prey refuge and delay

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HIGHLIGHTS

- A diffusive predator–prey system with a constant prey refuge and time delay is considered.
- The stability and Turing instability of the positive equilibrium are obtained.
- The existence of Hopf bifurcation at the positive equilibrium is obtained.
- The direction of the Hopf bifurcation and stability of the bifurcating periodic solutions are determined.

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ABSTRACT

In this paper, a diffusive predator–prey system with a constant prey refuge and time delay subject to Neumann boundary condition is considered. Local stability and Turing instability of the positive equilibrium are studied. The effect of time delay on the model is also obtained, including locally asymptotical stability and existence of Hopf bifurcation at the positive equilibrium. And the properties of Hopf bifurcation are determined by center manifold theorem and normal form theorem of partial functional differential equations. Some numerical simulations are carried out.

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1. Introduction

Dynamics in predator–prey system is one of the interesting subjects in ecology and mathematical ecology [1–5]. Some models have been studied from various point of views [6–11]. The dynamics of Leslie–Gower model and its various modifications have received great attention [12–18]. One class of modified Leslie–Gower

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models is:

$$\begin{cases} \dfrac{du}{dt} = r_1 u\left(1 - \dfrac{u}{K}\right) - \varphi(u)v, \\ \dfrac{dv}{dt} = r_2 v\left(1 - \dfrac{\beta v}{u+b}\right), \\ u(0) > 0, \qquad v(0) > 0, \end{cases} \tag{1.1}$$

where u and v represent the populations of prey and predator, respectively. All parameters involved with the model are positive. The prey grows logistically with carrying capacity K and intrinsic growth rate r_1 in the absence of predation. $\varphi(u)$ is functional response. The predators grow logistically with intrinsic growth rate r_2. The carrying capacity of the predator is proportional to the prey and other food.

Some researchers have studied system (1.1) with Holling type functional response [19–21]. In these studies, functional response is prey dependent and fail to model the interference among predators [22]. Skalski and Gilliam [23] pointed that three predator-dependent functional responses (Beddington–DeAngelis, Crowley–Martin, and Hassell–Varley) can provide better description of predator feeding over a range of predator–prey abundances present. In some cases, the Beddington–DeAngelis type preformed even better. Taking into account the inhomogeneous distribution of the predators and their preys in different spatial locations, Yang consider a modified Leslie–Gower with diffusion and Beddington–DeAngelis functional response [24]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = D_1 \Delta u + r_1 u\left(1 - \dfrac{u}{K}\right) - \dfrac{\alpha u v}{p + u + hv}, & x \in \Omega = (0, l\pi), \quad t > 0 \\ \dfrac{\partial v}{\partial t} = D_2 \Delta v + r_2 v\left(1 - \dfrac{\beta v}{u+b}\right), & x \in \Omega = (0, l\pi), \quad t > 0 \\ u_x(0,t) = v_x(0,t) = 0, \qquad u_x(l\pi, t) = v_x(l\pi, t) = 0, & t > 0, \\ u(x,0) = u_0(x) \geq 0, \qquad v(x,0) = v_0(x) \geq 0, & x \in \Omega = (0, l\pi), \end{cases} \tag{1.2}$$

where $l \in R^+$. For system (1.2), Yang [24] gives a sufficient condition to ensure persistence and sufficient conditions for the global asymptotical stability of the unique positive equilibrium by using a comparison method.

In [25], Kar pointed out that mite predator–prey interactions often exhibit spatial refugia that afford the prey some degree of protection from predation and reduce the chance of extinction due to predation. By virtue of this, many researchers have studied predator–prey system with prey refuge in recent years [26–29]. Prey refuge can be broadly defined to any strategy that can decrease predation rate. In fact, the effects of prey refuges on predator–prey model are very complex in nature, but for modeling purposes, it can be divided into two types: one type is to protect a constant number of prey and another type is to protect a constant fraction of prey. Some researchers have studied different predator–prey models with different functional responses and prey refuge. The most wildly reported conclusion is that a constant fraction and a constant amount prey refuges all can stabilize the interior equilibrium [30–32,29,33,34,27,35]. However, to our knowledge, there are surprisingly few conclusions in system (1.2) incorporating a constant number of prey refuge, which is motivated for our paper.

It is well known that time delay plays an important role in many biological dynamical systems, being particularly relevant into predator–prey models [6,13,36,20,37]. Time delays have been recognized to contribute critically to the stable or unstable outcome of prey densities due to predation. Time delays are incorporated in predator–prey models due to maturation time, capturing time, gestation time or other reasons. For modified Leslie–Gower model (1.1), the discrete delay has been incorporated in the negative feedback of the predator's density by many researchers [6,20,37,38]. For system (1.2), incorporate a constant number of prey refuge and discrete delay in the negative feedback of the predator's density, we propose the

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