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A class of hemivariational inequalities for nonstationary Navier–Stokes equations[☆]

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ABSTRACT

This paper is devoted to the study of a class of hemivariational inequalities for the time-dependent Navier–Stokes equations, including both boundary hemivariational inequalities and domain hemivariational inequalities. The hemivariational inequalities are analyzed in the framework of an abstract hemivariational inequality. Solution existence for the abstract hemivariational inequality is explored through a limiting procedure for a temporally semi-discrete scheme based on the backward Euler difference of the time derivative, known as the Rothe method. It is shown that solutions of the Rothe scheme exist, they contain a weakly convergent subsequence as the time step-size approaches zero, and any weak limit of the solution sequence is a solution of the abstract hemivariational inequality. It is further shown that under certain conditions, a solution of the abstract hemivariational inequality is unique and the solution of the abstract hemivariational inequality depends continuously on the problem data. The results on the abstract hemivariational inequality are applied to hemivariational inequalities associated with the time-dependent Navier–Stokes equations.

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1. Introduction

Variational inequalities and hemivariational inequalities each form an important family of nonlinear problems with applications in several fields such as mechanics, physics, engineering, and economics. One of the bases for its development was the contribution of Fichera [1] on the solution of the frictionless contact problem between a linearly elastic body and a rigid foundation, posed by Signorini [2]. Mathematical analysis of variational inequalities started in 1960s. The foundations of the mathematical theory of variational inequalities were laid in [3–6]. In particular, Stampacchia coined the term "Variational Inequality" in [3]. The study of variational inequalities and their applications was popularized by several early monographs, such as [7–9]. The monographs [10–12] provide comprehensive coverage of numerical methods and their analysis for solving various variational inequalities. Applications of variational inequalities in contact mechanics and plasticity can be found in [13–15].

The notion of hemivariational inequalities was first introduced by Panagiotopoulos in early 1980s [16] and is closely related to the development of the concept of the generalized gradient of a locally Lipschitz functional provided by Clarke [17,18]. Interest in hemivariational inequalities originated, similarly as in variational inequalities, in mechanical problems. From this point of view, the inequality problems in mechanics can be divided into two main classes: that of variational inequalities which is concerned with convex energy functionals (potentials), and that of hemivariational inequalities which is concerned with nonsmooth and nonconvex energy functionals (superpotentials). Through the formulation of hemivariational inequalities, problems involving nonmonotone, nonsmooth and multivalued constitutive laws, forces, and boundary conditions can be treated successfully, both theoretically and numerically. During the last three decades, hemivariational inequalities were shown to be very useful across a variety of subjects, and there is a large number of problems which lead to mathematical models expressed in terms of hemivariational inequalities. The mathematical literature dedicated to this field is growing rapidly. The theory, numerical solution and applications of hemivariational inequalities can be found in several monographs [19–22] and the references therein. Analysis of the finite element method for solving hemivariational inequalities can be found in the monograph [23]. In the recent papers [24,25], optimal order error estimates are derived, for the first time, for the linear finite element approximations of some hemivariational inequalities.

Time-dependent and time-independent Navier–Stokes equations have been research topics of substantial efforts in their mathematical theories, numerical solutions, computer simulations, and applications. In this regard, we refer the reader to [26,27] for mathematical theories and to [28] for numerical analysis of initial–boundary or boundary value problems of the Navier–Stokes equations. Starting with Ref. [29], variational inequalities for the Navier–Stokes equations or the Stokes equations are formulated and studied for viscous incompressible fluid flow problems involving leak or slip boundary conditions. Some recent references on the analysis and numerical solution of such variational inequalities include [30–34]. In the context of hemivariational inequalities associated with the Navier–Stokes equations, a stationary hemivariational inequality is studied in [35], and an evolutionary hemivariational inequality is studied in [36]; these two papers provide existence results to the hemivariational inequalities, as well as the solution uniqueness for the stationary hemivariational inequality. In this paper, we study hemivariational inequalities for the time-dependent Navier–Stokes equations, through a unified framework of an abstract problem. We explore the solution existence, uniqueness, and continuous dependence on the data for an abstract hemivariational inequality problem, and apply the results to the nonstationary hemivariational inequalities for the Navier–Stokes equations that are of the boundary type, corresponding to nonlinear slip boundary conditions, and of the domain type, corresponding to hydraulic flow controls.

The organization of the rest of the paper is as follows. In Section 2 we present some definitions and some auxiliary material. In Section 3, we introduce an abstract hemivariational inequality that includes as a particular case the hemivariational inequalities for the Navier–Stokes equations. The abstract hemivariational