Nonlinear Analysis: Real World Applications 31 (2016) 277–287

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

The effect of refuge and immigration in a predator–prey system in the presence of a competitor for the prey

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ARTICLE INFO

Article history:
Received 14 June 2015
Received in revised form 10 January 2016
Accepted 12 February 2016

Keywords:
Prey–predator
Refuge
Immigration
Stability
Bifurcation

ABSTRACT

This paper considers the effect of immigration and refuge on the dynamics of a three species system in which one predator feeds on one of two competing species. Immigration is assumed only for the species which is not attacked by the predator.

The main results address the stability of the system. Namely, it is shown that increasing the number of refuges stabilizes the system, whereas the opposite holds true by increasing the immigration rate. Also, one result about the persistence of the system and one concerning the global stability of the coexistence equilibrium are presented. Some numerical simulations illustrate the obtained results.

1. Introduction

The effect of predation on two competing populations has received much attention from the researchers [1–5] and has contributed to our understanding of competitive process. Theoretical studies on competing and invading species, niches and their role are discussed in [6,7]. More complicated situations arise when the predator exhibits a preference for a particular prey species and constant immigration of prey occurs. Due to predation pressure, the preferred prey try to hide themselves from predation.

The study of consequences of the refuge is a topic of interest in theoretical ecology. Although much work has been done, still many problems are unsolved. Hence, mathematical analysis of such systems requires the efforts of both mathematicians and ecologists.

In natural ecosystem, most of the animals try to hide from their predators. So they make refuge in different ways. The refuge habitat consists of burrows [8], trees [9], cliff faces [10], thick vegetation [11], etc. In aquatic ecosystem it is found that benthic coral cover provides the refuge for prey fish in pristine coral reefs [12].

Refuge has an important role in stability of prey–predator interactions [13]. Perhaps the first empirical demonstration of this effect was Gause's [14] experiments with protozoan predator *Didinium* feeding on

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http://dx.doi.org/10.1016/j.nonrwa.2016.02.004

Paramecium. In the case of *Paramecium–Didinium* system, the presence of refuge allowed some of the *Paramecium* to avoid predation, the prey population persisted after all of the predators had starved. This is likely to be a case where a constant proportion of the prey is protected by the refuge, a situation that is believed to have little or no effect on the stability of the prey population [15]. Due to fixed proportion refuge, the death rate does not increase with population density and so the presence of refuge does not cause any negative feedback effect, a necessary condition for stabilization [16]. But proportional refuges can enhance the persistence of the populations. When the refuge shelters a constant number of individuals, the prey mortality rate increases with population density beyond the capacity of the refuge and this will cause negative feedback stabilizing the system. An example of a constant refuge is the gypsy moth (Lymantria dispar), which consumes bark flaps and any other suitable object, as a protection against predation by mice and birds [17].

Refuge on prey–predator system has been studied in [18–26]. In case of three dimensional system, the effect of refuge on prey that has a competitor species is not investigated so far [27–29]. Thus our main aim in this article is to investigate the effect of refuge and immigration on a predator–prey system with competitor for the prey. Ultimately we will show that this type of modelling enhances the ecological stability.

The organization of this paper is as follows. In Section 2, we introduce our mathematical model with constant prey refuge. In Section 3, we analyse our model with regard to equilibria and their stabilities. Bifurcation phenomenon is established in Section 4. Persistence criterion is developed in Section 5. Numerical examples are presented in Section 6. A brief discussion is presented in Section 7.

2. Mathematical model

We consider a habitat where prey and predator are living together incorporating constant prey refuges. Immigration is assumed only for the species which is not attacked by the predator. The prey has a competitor. The predator feeds on one of two competing species. The model is described by

$$\begin{aligned} \frac{dx_1}{dt} &= x_1(r_1 - a_{11}x_1 - a_{12}x_2) - c_1 f(x_1) y \\ \frac{dx_2}{dt} &= x_2(r_2 - a_{21}x_1 - a_{22}x_2) + h \\ \frac{dy}{dt} &= y(-d + c_2 f(x_1)) \end{aligned} \tag{1}$$

where

$$f(x_1) = \begin{cases} 0 & x_1 \le m \\ x_1 - m & x_1 > m \end{cases}$$

with initial conditions given by $x_i(0) = x_{i0} > 0,\ i = 1, 2, y(0) = y_0 > 0$. Biological significance of the symbols in system (1) are given below.

- $x_1(t)$ density of the prey
- $x_2(t)$ density of the competitor for the prey
- $y(t)$ the density of predator
- a_{11} the intraspecific competition coefficient of the prey
- a_{12} the interspecific competition coefficient of the competitor for the prey
- a_{21} the interspecific competition coefficient of the prey
- a_{22} the intraspecific competition coefficient of the competitor for the prey
- m the constant number of prey using refuge
- h the constant number of competitor for the prey immigration
- c_1 the per capita predator consumption rate